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ON THE EDGE VERSION OF GEOMETRIC-ARITHMETIC INDEX

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The edge version of geometric–arithmetic index of graphs is introduced based on the end-vertex degrees of edges of their line graphs and some properties especially lower and upper bounds and the relation between vertex and edge versions are obtained.

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1. Introduction

A single number that can be used to characterize some property of the graph of a molecule is called a topological index for that graph. There are numerous topological descriptors that have found some applications in theoretical chemistry, especially in QSPR/QSAR research [1]. The oldest topological index which introduced by Harold Wiener in 1947 is ordinary (vertex) version of Wiener index [2] which is the sum of all distances between vertices of a graph. Also, the edge versions of Wiener index which were based on distance between edges introduced by Iranmanesh et al. in 2008 [3].

One of the most important topological indices is the well-known branching index introduced by Randic [4] which is defined as the sum of certain bond contributions calculated from the vertex degree of the hydrogen suppressed molecular graphs.

Motivated by the definition of Randic connectivity index based on the end-vertex degrees of edges in a graph connected G with the vertex set $V(G)$ and the edge set $E(G)$ [5,6], Vukicevic and Furtula [7] proposed a topological index named the geometric-arithmetic index (shortly GA) as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

where d_u denotes the degree of the vertex u in *G*. The reader can find more information about geometric-arithmetic index in [7-9].

It is natural which we introduce the edge version of geometric-arithmetic index based on the end-vertex degrees of edges in a line graph of G as follows

$$GA(G) = \sum_{ef \in E(L(G))} \frac{2\sqrt{d_e d_f}}{d_e + d_f}$$

where d_e denotes the degree of the edge e in *G*.

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In this paper, we focus our attention to this index and its main results including lower and upper bounds.

2. The Main Results

Firstly, we compute the edge GA index of some familiar graphs P_n , S_n , K_n , $K_{m,n}$ and C_n which are the path, star, complete graph , complete bipartite graph and cycle, respectively. Also, we compute it for $TUC_4C_8(S)$ nanotubes.

Example 1. The edge GA index of some familiar graphs P_n , S_n , K_n , $K_{m,n}$ and C_n is

1. $GA_e(P_n) = GA(P_{n-1}) = \frac{4\sqrt{2}}{3} + (n-4)$
2. $GA_e(S_n) = GA(K_{n-1}) = \binom{n-1}{2}$
3. $GA_e(K_n) = GA((2n-2) - regular\ graph) = \frac{n(n-1)^2}{2}$
4. $GA_e(K_{m,n}) = GA((m+n) - regular\ graph) = \frac{mn(m+n)}{2}$
5. $GA_e(C_n) = GA(C_n) = n$

Now, we compute the edge GA index of $TUC_4C_8(S)$ nanotubes. In Figure 1, the $TUC_4C_8(S)$ nanotube is shown.

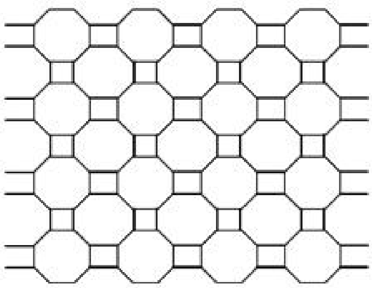

Fig. 1. Two dimensional lattice of $TUC_4C_8(S)$ nanotube, $p = 4, q = 4$.

We denote the number of squares in one row of squares by p and the number of rows of squares by q. For example see the Figure 3. We show the molecular graph of $TUC_4C_8(S)$ nanotube with TUC in following computation.

Example 2. The edge GA index of $TUC_4C_8(S)$ nanotube is

$$GA_e(TUC) = 24pq + \frac{8p\sqrt{6}}{5} + \frac{16p\sqrt{12}}{7} - 20p$$

Proof. Consider the $TUC_4C_8(S)$ nanotube. The number of edges of line graph of $TUC_4C_8(S)$ nanotube is $24pq - 8p$. The number of edges which have the vertices with degrees 2 and 3 in line graph $TUC_4C_8(S)$ nanotube is $4p$. Also, The number of edges which have the vertices with degrees 3 and 4 in line graph $TUC_4C_8(S)$ nanotube is $8p$. In addition, the other edges which their number is $24pq - 20p$ have the vertices with degrees 4. Therefore, the desire result can be concluded.

In following, we will find some bounds for edge GA index.

Proposition 3. Let *G* be a connected graph with *n* vertices and *m* edges. Therefore, we have

$$GA_e(G) \le \sum_{x\in V(G)} {d_x}^2 - 2m$$

where d_x is the degree of vertex $x \in V(G)$.

Proof. Consider a connected graph *G* with *n* vertices and *m* edges. Due to the fact that the geometric mean is less than or equal to the arithmetic mean, we have for edge GA index of graph *G*,

$$GA_e(G) \le |E(L(G))| = \sum_{x\in V(G)} {d_x}^2 - 2m.$$

Theorem 4. Let *G* be a simple graph with *n* vertices and *m* edges, then

$$0 \le GA_e(G) \le \frac{n(n-1)^2}{2}$$

Lower bound is achieved if and only if *G* is an empty graph and upper bound is achieved if and only if *G* is a complete graph.

Proof. Since $\frac{2\sqrt{d_e d_f}}{d_e + d_f}$ is positive for each edge of line graph of G. Then, $GA_e(G) \ge 0$ and if G is empty, $GA_e(G) = 0$. Also, Due to the fact that the geometric mean is less than or equal to the arithmetic mean, we have

$$GA_e(G) \le 1.|E(L(G))| \le 1.\binom{n}{2}(n-2).$$

And if G is complete graph, $GA_e(G) = |E(L(G))| = \binom{n}{2}(n-2)$. □

Theorem 5. Let *G* be a simple graph with *n* vertices and *m* edges, then

$$(n-4) + \frac{4\sqrt{2}}{3} \le GA_e(G) \le \frac{n(n-1)^2}{2}$$

Lower bound is achieved if and only if *G* is a path P_n and upper bound is achieved if and only if *G* is a complete graph.

Proof. The upper bound follows from Theorem 4. Now, we prove the lower bound. As we know from vertex version of GA index, the acyclic graphs have the least vertex GA index. Therefore, the only graph which its line graph is acyclic is path. Then, the edge GA index has its least value for paths. Hence,

$$GA_e(G) \ge GA_e(P_n) = (n-4) + \frac{4\sqrt{2}}{3}.$$

Nordhaus and Guddum [10]gave bounds for the sum of chromatic numbers of a graph and its complement. Nordhaus-Gaddum-type results for many graph invariants are known. Here we give Nordhaus-Gaddum-type result for the edge GA index.

Result 6. Let G be a simple graph with n vertices and m edges, then the Nordhaus-Gaddum-type result for edge GA index of G is

$$\frac{3(n^2-n-4)+8\sqrt{2}}{6} < GA_e(G)+GA_e(\bar{G}) \leq \frac{(n-2)(n-1)n(n+1)}{8}$$

Proof. According to Theorem 5, we have $GA_e(G) \geq m-2+\frac{4\sqrt{2}}{3}$. Therefore, by replacing $\binom{n}{2}$ instead of m in last equation, the lower bound is concluded.

Also, according to Proposition 3, we have $GA_e(G) \leq |E(L(G))| \leq \binom{m}{2}$. Then by replacing $\binom{n}{2}$ instead of m in last equation, the upper bound is concluded. □

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