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COMPUTING THE SECOND- AND THIRD- CONNECTIVITY INDEX OF AN INFINITE CLASS OF DENDRIMER NANOSTARS

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The m-connectivety index of a graph G is defined to be $^{m}\chi(G)=\sum_{v_{i_1}v_{i_2}..v_{i_{m+1}}}\frac{1}{\sqrt{d_{i_1}d_{i_2}..d_{i_{m+1}}}}$.

where $v_{i_1}v_{i_2}...v_{i_{m+1}}$ runs over all paths of length m in G and d_i is the degree of vertex v_i. In this paper, we give explicit formulas for the second- and third- order connectivity index of an infinite class of dendrimer nanostars.

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1. Introduction

Let G be a connected simple graph. The m-connectivity index of G is defined as

$$^{m}\chi(G)=\sum_{v_{i_1}v_{i_2}\dots v_{i_{m+1}}}\frac{1}{\sqrt{d_{i_1}d_{i_2}...d_{i_{m+1}}}}$$

where $v_{i_1}v_{i_2}...v_{i_{m+1}}$ runs over all paths of length m in G and d_i is the degree of vertex v_i. In particular, 2-connectivity and 3-connectivity index are defined as

$$^{2}\chi(G)=\sum_{v_{i_1}v_{i_2}v_{i_3}}\frac{1}{\sqrt{d_{i_1}d_{i_2}d_{i_3}}}$$

and

$$^{3}\chi(G)=\sum_{v_{i_1}v_{i_2}v_{ii_3}v_{i_4}}\frac{1}{\sqrt{d_{i_1}d_{i_2}d_{i_3}d_{i_4}}},$$

respectively.

During the past several decades, there are many papers dealing with the connectivity index.

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The reader may consult [1-11] and references cited therein.
In this paper, we shall give explicit computing formulas for 2- and 3- connectivty of a type of dendrimer nanostars.

2. Main results

Let NS[n] denote a kind of dendrimer nanostars with n growth stages, see for example, Fig.s 1 and 2.

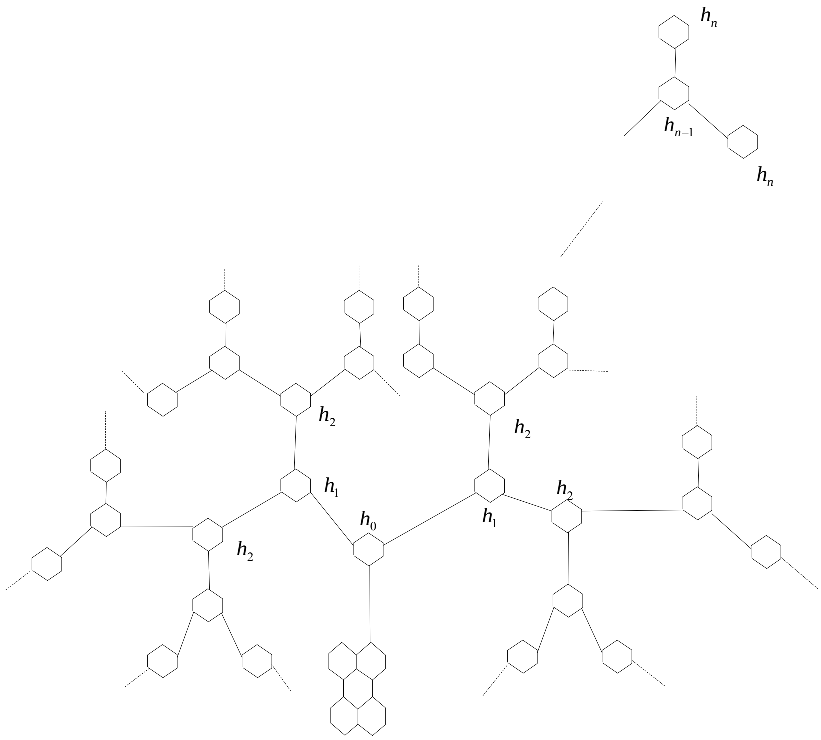

Fig. 1. The dendrimer nanostar NS[n]

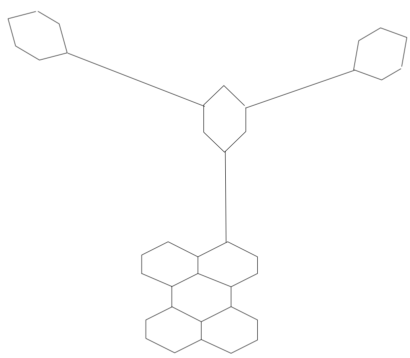

Fig. 2. The dendrimer nanostar NS[1]

In the following, we shall compute the second- and third- order connectivity index for the dendrimer nanostars as shown in Fig. 1.

We first give an exact formula of the second-order connectivity index for this dendrimer nanostar.

Theorem 1. *Let* NS[n] *be the dendrimer nanostar as shown in Fig. 1. Then*

$$^{2}\chi(NS[n]) = 12\sqrt{2} + \frac{25\sqrt{3}}{3} + \left(\frac{31}{6}\sqrt{2} + 2\sqrt{3}\right)(2^{n-1} - 1)$$

Proof. Let d_{ijk} denote the number of 2 paths whose three consecutive vertices are of degree $i,\ j,\ k$, respectively. Also, we use $d_{ijk}^{(s)}$ to mean d_{ijk} in the s^{th} stage. Obviously, $d_{ijk}^{(s)} = d_{kji}^{(s)}$.

Firstly, we compute the value of $^{2}\chi(NS[1])$. It is easily seen that

$$d_{222}^{(1)} = 6,\ d_{223}^{(1)} = 12,\ d_{232}^{(1)} = 6,\ d_{233}^{(1)} = 24,\ d_{323}^{(1)} = 3,\ d_{333}^{(1)} = 12.$$

So we have

$$^{2}\chi(NS[1]) = \frac{6}{\sqrt{2\times2\times2}} + \frac{12}{\sqrt{2\times2\times3}} + \frac{6}{\sqrt{2\times3\times2}} + \frac{24}{\sqrt{2\times3\times3}} + \frac{3}{\sqrt{3\times2\times3}} + \frac{12}{\sqrt{3\times3\times3}}$$

$$= \frac{3\sqrt{2}}{2} + 6\sqrt{3} + \sqrt{3} + 4\sqrt{2} + \frac{\sqrt{2}}{2} + \frac{4\sqrt{3}}{3}$$

$$= 12\sqrt{2} + \frac{25\sqrt{3}}{3}.$$

Now, we are ready to deduce the relation between $^{2}\chi(NS[s])$ and $^{2}\chi(NS[s-1])$ for $s \geq 2.$

$$d_{222}^{(s)} = d_{222}^{(s-1)} + 3\cdot2^{s} - 3\cdot2^{s-1} = d_{222}^{(s-1)} + 3\cdot2^{s-1}.$$

$$d_{223}^{(s)} = d_{223}^{(s-1)} + 2\cdot2^{s} - 2\cdot2^{s-1} = d_{223}^{(s-1)} + 2^{s}.$$

$$d_{232}^{(s)} = d_{232}^{(s-1)} + 2^{s} + 2\cdot2^{s-1} = d_{232}^{(s-1)} + 2^{s+1}.$$

$$d_{233}^{(s)} = d_{233}^{(s-1)} + 2\cdot2^{s} + 4\cdot2^{s-1} = d_{232}^{(s-1)} + 2^{s+2}.$$

$$d_{323}^{(s)} = d_{323}^{(s-1)} + 3\cdot2^{s-1}.$$

Obviously, for any $(i, j, k) \neq (2,2,2),(2,2,3),(2,3,2),(2,3,3),(3,2,3),$ we have $d_{ijk}^{(s)} = 0$ or $d_{ijk}^{(s)} = d_{ijk}^{(s-1)} = \cdots = d_{ijk}^{(1)}$ for $s = 2,\cdots,n.$

Thus,

$$^{2}\chi(NS[n]) = {}^{2}\chi(NS[n-1]) + \frac{3\cdot 2^{n-1}}{\sqrt{2\times 2\times 2}} + \frac{2^{n}}{\sqrt{2\times 2\times 3}} + \frac{2^{n+1}}{\sqrt{2\times 3\times 2}} + \frac{2^{n+2}}{\sqrt{2\times 3\times 3}} + \frac{3\cdot 2^{n-1}}{\sqrt{3\times 2\times 3}}$$

$$= {}^{2}\chi(NS[n-1]) + \frac{3\cdot 2^{n-2}}{\sqrt{2}} + \frac{2^{n-1}}{\sqrt{3}} + \frac{2^{n}}{\sqrt{3}} + \frac{2^{n+2}}{3\sqrt{2}} + \frac{2^{n-1}}{\sqrt{2}}$$

$$= {}^{2}\chi(NS[n-1]) + (2\sqrt{3} + \frac{31\sqrt{2}}{6})2^{n-2}.$$

By the above recursive formula for $^{2}\chi(NS[n])$, we obtain

$$^{2}\chi(NS[n]) = {}^{2}\chi(NS[n-1]) + (2\sqrt{3} + \frac{31\sqrt{2}}{6})2^{n-2}$$

$$= {}^{2}\chi(NS[n-2]) + (2\sqrt{3} + \frac{31\sqrt{2}}{6})(2^{n-2} + 2^{n-3})$$

$$= \cdots$$

$$= {}^{2}\chi(NS[1]) + (2\sqrt{3} + \frac{31\sqrt{2}}{6})(2^{n-2} + 2^{n-3} + \cdots + 1)$$

$$= 12\sqrt{2} + \frac{25\sqrt{3}}{3} + (2\sqrt{3} + \frac{31\sqrt{2}}{6})(2^{n-1} - 1).$$

Next, we shall give an exact formula of the third-order connectivity index for the dendrimer nanostar as shown in Fig. 1.

Theorem 2. *Let* NS[n] *be the dendrimer nanostar as shown in Fig. 1. Then*

$$^{3}\chi(NS[n]) = \frac{155}{18} + \frac{45\sqrt{6}}{18} + (5 + \frac{4\sqrt{6}}{3})(2^{n-1} - 1).$$

Proof. Let d_{ijkl} denote the number of 3 paths whose four consecutive vertices are of degree $i,\ j,\ k,\ l$, respectively. Also, we use $d_{ijkl}^{(s)}$ to mean d_{ijkl} in the s^{th} stage. Obviously, $d_{ijkl}^{(s)} = d_{lkji}^{(s)}$.

Firstly, we compute the value of $^{3}\chi(NS[1])$.

It is easy to obtain the following:

$d_{2222}^{(1)} = 4$, $d_{2223}^{(1)} = 10$, $d_{2232}^{(1)} = 6$, $d_{2233}^{(1)} = 18$, $d_{2332}^{(1)} = 12$,

$d_{2333}^{(1)} = 17$, $d_{3223}^{(1)} = 1$, $d_{3232}^{(1)} = 6$, $d_{3233}^{(1)} = 4$, $d_{3333}^{(1)} = 13$.

So we have

$$^{3}\chi(NS[1]) = \frac{4}{\sqrt{2\times 2\times 2\times 2}} + \frac{10}{\sqrt{2\times 2\times 2\times 3}} + \frac{6}{\sqrt{2\times 2\times 3\times 2}} + \frac{18}{\sqrt{2\times 2\times 3\times 3}} +$$
$$\frac{12}{\sqrt{2\times 3\times 3\times 2}} + \frac{17}{\sqrt{2\times 3\times 3\times 3}} + \frac{1}{\sqrt{3\times 2\times 2\times 3}} + \frac{6}{\sqrt{3\times 2\times 3\times 2}} +$$
$$\frac{4}{\sqrt{3\times 2\times 3\times 3}} + \frac{13}{\sqrt{3\times 3\times 3\times 3}}$$
$$= \frac{155}{18} + \frac{45\sqrt{6}}{18}.$$

Now, we are ready to deduce the relation between $^{3}\chi(NS[s])$ and $^{3}\chi(NS[s-1])$ for $s \geq 2.$

$$d_{2222}^{(s)} = d_{2222}^{(s-1)} + 2\cdot 2^{s} - 2\cdot 2^{s-1} = d_{2222}^{(s-1)} + 2^{s}.$$
$$d_{2223}^{(s)} = d_{2223}^{(s-1)} + 2\cdot 2^{s} - 2\cdot 2^{s-1} = d_{2223}^{(s-1)} + 2^{s}.$$
$$d_{2232}^{(s)} = d_{2232}^{(s-1)} + 2\cdot 2^{s} - 2\cdot 2^{s-1} = d_{2232}^{(s-1)} + 2^{s}.$$

$$d_{2233}^{(s)} = d_{2233}^{(s-1)} + 2\cdot 2^{s} - 2\cdot 2^{s-1} = d_{2233}^{(s-1)} + 2^{s}.$$

$$d_{2323}^{(s)} = d_{2323}^{(s-1)} + 6\cdot 2^{s-1} = d_{2323}^{(s-1)} + 3\cdot 2^{s}.$$
$$d_{2332}^{(s)} = d_{2332}^{(s-1)} + 4\cdot 2^{s} - 4\cdot 2^{s-1} = d_{2332}^{(s-1)} + 2^{s+1}.$$
$$d_{3233}^{(s)} = d_{3233}^{(s-1)} + 6\cdot 2^{s-1} = d_{3233}^{(s-1)} + 3\cdot 2^{s}.$$

Obviously, for any $(i, j, k, l) \neq (2,2,2,2),(2,2,2,3),(2,2,3,2),(2,2,3,3),(2,3,2,3),$ $(2,3,3,2),(3,2,2,3),$ we have $d_{ijkl}^{(s)} = 0$ or $d_{ijkl}^{(s)} = d_{ijkl}^{(s-1)} = \cdots = d_{ijkl}^{(1)}$ for $s = 2,\cdots,n.$

Thus,

$$^{3}\chi(NS[n]) = {}^{3}\chi(NS[n-1]) + \frac{2^{n}}{\sqrt{2\times2\times2\times2}} + \frac{2^{n}}{\sqrt{2\times2\times2\times3}} + \frac{2^{n}}{\sqrt{2\times2\times3\times2}} +$$

$$\frac{2^{n}}{\sqrt{2\times2\times3\times3}} + \frac{3\cdot2^{n}}{\sqrt{2\times3\times2\times3}} + \frac{2^{n+1}}{\sqrt{2\times3\times3\times2}} + \frac{3\cdot2^{n}}{\sqrt{3\times2\times3\times3}}$$

$$= {}^{3}\chi(NS[n-1]) + 2^{n-2} + (\frac{4}{3} + \frac{2}{\sqrt{6}})\cdot2^{n-1} + (\frac{1}{3} + \frac{1}{\sqrt{6}})\cdot2^{n}$$

$$= {}^{3}\chi(NS[n-1]) + (5 + \frac{4\sqrt{6}}{3})2^{n-2}.$$

By the above recursive formula for $^{3}\chi(NS[n])$, we obtain

$$^{3}\chi(NS[n]) = {}^{3}\chi(NS[n-1]) + (5 + \frac{4\sqrt{6}}{3})\cdot2^{n-2}$$

$$= {}^{3}\chi(NS[n-2]) + (5 + \frac{4\sqrt{6}}{3})(2^{n-2} + 2^{n-3})$$

$$= \cdots$$

$$= {}^{3}\chi(NS[1]) + (5 + \frac{4\sqrt{6}}{3})(2^{n-2} + 2^{n-3} + \cdots + 1)$$

$$= \frac{155}{18} + \frac{45\sqrt{6}}{18} + (5 + \frac{4\sqrt{6}}{3})(2^{n-1} - 1).$$

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