THE THIRD GEOMETRIC-ARITHMETIC INDEX OF $TUC_4C_8(S)$ NANOTORUS

JIANXIU HAO*
Institute of Mathematics, Physics and Information Sciences,
Zhejiang Normal University, P. O. Box: 321004,
Jinhua, Zhejiang, P.R. China

The third geometric-arithmetic index is an important topological index in mathematical chemistry. In this paper we study the third geometric-arithmetic index of $TUC_4C_8(S)$ nanotorus.

(Received September 8, 2012; September 28, 2012)

Keywords: The third geometric-arithmetic index, $TUC_4C_8(S)$, nanotori.

1. Introduction

The third geometric-arithmetic index is introduced by B. Zhou, I. Gutman, B. Furt--ula, Z. Du[1, 2]. It is defined as follows [1, 2]: for a simple connected graph G,

$GA_3(G) = \sum_{uv \in E(G)} \frac{\sqrt{m_u . m_v}}{0.5(m_u + m_v)}$, where m_u is defined as follows: let x be a vertex and uv be an edge of graph G, the distance between x and uv is defined as follows: d(x, uv) = min{ d(x, u), d(x, v) }, where d(x, u) is the length of the shortest path that connects x and u in G. For uv∈E(G), let m_u = |{ f∈E(G): d(u, f) < d(v, f) }|. GA_3 index is a possible tool for QSAR/QSPR researches and it gives somewhat better predictions than those of GA_2 does [1, 2].

In this paper we study the third geometric-arithmetic index of $TUC_4C_8(S)$ nanotorus. For the figure of $TUC_4C_8(S)$ nanotorus, see [3].

2. Main result

Theorem 2.1. *Let G be $TUC_4C_8(S)[p, q]$ nanotorus, where p≥2, q≥2, we have*

$$GA_3(G) = 12pq.$$

Proof. In the following, let q≥3. Firstly, we label the levels of G from bottom to top with 1, 2, …, 2q respectively. Secondly, we label the vertices in level i with x_{i1}, x_{i2}, …, $x_{i,4p}$, where i = 1, 2, …, 2q. Clearly, the edge number of G is 12pq. By the symmetry of p and q, in the following let p≥q.

*Corresponding author: sx35@zjnu.cn

Case 1. e = $x_{14}x_{15}$.

Clearly, the edges in A are equidistant from x_{14} and x_{15}, where

$$A = \{x_{14}x_{15}, x_{24}x_{25}, x_{34}x_{35}, \ldots, x_{2q,4}x_{2q,5};$$
$$x_{1,2p+4}x_{1,2p+5}, x_{2,2p+4}x_{2,2p+5}, x_{3,2p+4}x_{3,2p+5}, \ldots, x_{2q,2p+4}x_{2q,2p+5}\}.$$

When we delete the edges in A, we obtain two graphs A_1 and A_2 from G. Without loss of generality, let $x_{14} \in V(A_1)$ and $x_{15} \in V(A_2)$. Obviously, $m_{x_{14}} = |E(A_1)|$, $m_{x_{15}} = |E(A_2)|$. Thus, we have

$$m_{x_{14}} = m_{x_{15}} = 6pq - 2q.$$

Hence, we have

$$\frac{\sqrt{m_{x_{14}} m_{x_{15}}}}{0.5(m_{x_{14}} + m_{x_{15}})} = 1.$$

Case 2. e = $x_{12}x_{13}$.

Clearly, the edges in B are equidistant from x_{12} and x_{13}, where

$$B = \{x_{12}x_{13}, x_{22}x_{23}, x_{32}x_{33}, \ldots, x_{2q,2}x_{2q,3};$$
$$x_{1,2p+2}x_{1,2p+3}, x_{2,2p+2}x_{2,2p+3}, x_{3,2p+2}x_{3,2p+3}, \ldots, x_{2q,2p+2}x_{2q,2p+3}\}.$$

When we delete the edges in B, we obtain two graphs B_1 and B_2 from G. Without loss of generality, let $x_{12} \in V(B_1)$ and $x_{13} \in V(B_2)$. Obviously, $m_{x_{12}} = |E(B_1)|$, $m_{x_{13}} = |E(B_2)|$. Thus, we have

$$m_{x_{12}} = m_{x_{13}} = 6pq - 2q.$$

Hence, we have

$$\frac{\sqrt{m_{x_{12}} m_{x_{13}}}}{0.5(m_{x_{12}} + m_{x_{13}})} = 1.$$

Case 3. e = $x_{22}x_{32}$.

Subcase 3.1. q is odd.

Clearly, the edges in C are equidistant from x_{22} and x_{32}, where

$$C = \{x_{22}x_{32}, x_{23}x_{33}, x_{26}x_{36}, x_{27}x_{37}, \ldots, x_{2,4p-2}x_{3,4p-2}, x_{2,4p-1}x_{3,4p-1};$$
$$x_{q+2,1}x_{q+3,1}, x_{q+2,4}x_{q+3,4}, x_{q+2,5}x_{q+3,5}, x_{q+2,8}x_{q+3,8}, \ldots, x_{q+2,4p-3}x_{q+3,4p-3}, x_{q+2,4p}x_{q+3,4p}\}.$$

When we delete the edges in C, we obtain two graphs C_1 and C_2 from G. Without loss of generality, let $x_{22} \in V(C_1)$ and $x_{32} \in V(C_2)$. Obviously, $m_{x_{22}} = |E(C_1)|$, $m_{x_{32}} = |E(C_2)|$. Thus, we have

$$m_{x_{22}} = m_{x_{32}} = 6pq - 2p.$$

Hence, we have

$$\frac{\sqrt{m_{x_{22}} m_{x_{32}}}}{0.5(m_{x_{22}} + m_{x_{32}})} = 1.$$

Subcase 3.2. q is even.

Clearly, the edges in D are equidistant from x_{22} and x_{32}, where

$$D = \{x_{22}\,x_{32},\ x_{23}\,x_{33},\ x_{26}\,x_{36},\ x_{27}\,x_{37},\ \ldots,\ x_{2,4p-2}\,x_{3,4p-2},\ x_{2,4p-1}\,x_{3,4p-1};$$
$$x_{q+2,2}\,x_{q+3,2},\ x_{q+2,3}\,x_{q+3,3},\ x_{q+2,6}\,x_{q+3,6},\ x_{q+2,7}\,x_{q+3,7},\ \ldots,\ x_{q+2,4p-2}\,x_{q+3,4p-2},\ x_{q+2,4p-1}\,x_{q+3,4p-1}\}.$$

When we delete the edges in D, we obtain two graphs D_1 and D_2 from G. Without loss of generality, let $x_{22} \in V(D_1)$ and $x_{32} \in V(D_2)$. Obviously, $m_{x_{22}} = |E(D_1)|$, $m_{x_{32}} = |E(D_2)|$. Thus, we have

$$m_{x_{22}} = m_{x_{32}} = 6pq - 2p.$$

Hence, we have

$$\frac{\sqrt{m_{x_{22}} m_{x_{32}}}}{0.5(m_{x_{22}} + m_{x_{32}})} = 1.$$

Case 4. $e = x_{11}\,x_{21}$.

Subcase 4.1. q is odd.

Clearly, the edges in F are equidistant from x_{11} and x_{21}, where

$$F = \{x_{11}\,x_{21},\ x_{14}\,x_{24},\ x_{15}\,x_{25},\ x_{18}\,x_{28},\ \ldots,\ x_{1,4p-3}\,x_{2,4p-3},\ x_{1,4p}x_{2,4p};$$
$$x_{q+1,2}\,x_{q+2,2},\ x_{q+1,3}\,x_{q+2,3},\ x_{q+1,6}\,x_{q+2,6},\ x_{q+1,7}\,x_{q+2,7},\ \ldots,\ x_{q+1,4p-2}\,x_{q+2,4p-2},\ x_{q+1,4p-1}\,x_{q+2,4p-1}\}.$$

When we delete the edges in F, we obtain two graphs F_1 and F_2 from G. Without loss of generality, let $x_{11} \in V(F_1)$ and $x_{21} \in V(F_2)$. Obviously, $m_{x_{11}} = |E(F_1)|$, $m_{x_{21}} = |E(F_2)|$. Thus, we have

$$m_{x_{11}} = m_{x_{21}} = 6pq - 2p.$$

Hence, we have

$$\frac{\sqrt{m_{x_{11}} m_{x_{21}}}}{0.5(m_{x_{11}} + m_{x_{21}})} = 1.$$

Subcase 4.2. q is even.

Clearly, the edges in H are equidistant from x_{11} and x_{21}, where

$$H = \{x_{11}\,x_{21},\ x_{14}\,x_{24},\ x_{15}\,x_{25},\ x_{18}\,x_{28},\ \ldots,\ x_{1,4p-3}\,x_{2,4p-3},\ x_{1,4p}x_{2,4p};$$
$$x_{q+1,1}\,x_{q+2,1},\ x_{q+1,4}\,x_{q+2,4},\ x_{q+1,5}\,x_{q+2,5},\ x_{q+1,8}\,x_{q+2,8},\ \ldots,\ x_{q+1,4p-3}\,x_{q+2,4p-3},\ x_{q+1,4p}\,x_{q+2,4p}\}.$$

When we delete the edges in H, we obtain two graphs H_1 and H_2 from G. Without loss of generality, let $x_{11} \in V(H_1)$ and $x_{21} \in V(H_2)$. Obviously, $m_{x_{11}} = |E(H_1)|$, $m_{x_{21}} = |E(H_2)|$. Thus, we have

$$m_{x_{11}} = m_{x_{21}} = 6pq - 2p.$$

Hence, we have

$$\frac{\sqrt{m_{x_{11}} m_{x_{21}}}}{0.5(m_{x_{11}} + m_{x_{21}})} = 1.$$

Case 5. $e = x_{11}\,x_{12}$.

Clearly, the edges in I are equidistant from x_{11} and x_{12}, where

$$I = \{x_{11}\, x_{12},\, x_{23}\, x_{24},\, x_{35}\, x_{36}, \ldots, x_{q,2q-1}x_{q,2q};$$
$$x_{q+1,1}\, x_{q+1,2},\, x_{q+1,3}\, x_{q+1,4},\, x_{q+1,5}\, x_{q+1,6}, \ldots, x_{q+1,2q-1}\, x_{q+1,2q};$$
$$x_{1,2p+1}\, x_{1,2p+2},\, x_{2,2p+1}\, x_{2,2P+2},\, x_{3,2p+1}\, x_{3,2p+2}, \ldots, x_{2q,2p+1}x_{2q,2p+2};$$
$$x_{q+1,4p-2q+3}\, x_{q+1,4p-2q+4},\, x_{q+1,4p-2q+5}\, x_{q+1,4p-2q+6}, \ldots, x_{q+1,4p-3}\, x_{q+1,4p-2},\, x_{q+1,4p-1}\, x_{q+1,4p};$$
$$x_{q+2,4p-2q+3}\, x_{q+2,4p-2q+4},\, x_{q+3,4p-2q+5}\, x_{q+3,4p-2q+6}, \ldots, x_{2q-1,4p-3}\, x_{2q-1,4p-2},\, x_{2q,4p-1}\, x_{2q,4p}\}.$$

When we delete the edges in I, we obtain two graphs I_1 and I_2 from G. Without loss of generality, let $x_{11} \in V(I_1)$ and $x_{12} \in V(I_2)$. Obviously, $m_{x_{11}} = |E(I_1)|$, $m_{x_{12}} = |E(I_2)|$. Thus, we have

$$m_{x_{11}} = m_{x_{12}} = 6pq - 3q + 1.$$

Hence, we have

$$\frac{\sqrt{m_{x_{11}} m_{x_{12}}}}{0.5(m_{x_{11}} + m_{x_{12}})} = 1.$$

By the definition of $GA_3(G)$, when $q \geq 3$, the theorem follows. When $q = 2$, we can prove the theorem similarly.

Remark: let $p_1 = 2$, $q_1 = 6$, $p_2 = 3$, $q_2 = 4$, we have $GA_3(TUC_4C_8(S)[p_1, q_1]) = GA_3(TUC_4C_8(S)[p_2, q_2])$. Hence, the third geometric-arithmetic index is not good enough.

Acknowledgements

The project supported by the Natural Science Foundation of Department of Education of Zhejiang Province of China (No. 20070441); National Nature Science Foundation of China (No. 10971198), Zhejiang Provincial Natural Science Foundation of China (No. Y6090699); Zhejiang Innovation Project (No. T200905); Innovation Project of Optimization and Control of Network Systems of Zhejiang Normal University；ZSDZZZZXK03; Z6110786.

References

[1] B. Zhou, I. Gutman, B. Furtula, Z. Du, On two types of geometric-arithmetic index, Chem. Phys. Lett. **482,** 153-155 (2009).

[2] K.C. Das, I. Gutman, B. Furtula, Survey on geometric-arithmetic indices of graphs, MATCH Commun. Math. Comput. Chem. **65,** 595-644. (2011).

[3] L. Xu, H. Deng, PI indices of tori $T_{p,q}[C_4, C_8]$ covered by C_4 and C_8，MATCH Commun. Math. Comput. Chem. **57,** 485-502 (2007)