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ANALYTICAL SOLUTION OF INTERACTIVE DAMPING AND LONGITUDINAL AND NORMAL CONTACT STIFFNESS ON SENSITIVITY OF VIBRATION MODES OF RECTANGULAR AFM CANTILEVERS

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The interactive damping sensitivity and the resonant frequency of normal vibration and longitudinal vibration of an atomic force microscope (AFM) rectangular cantilever have been analyzed. Surface electrostatic attraction between the atoms in the tip of the probe and those in the surface is simulated with flexural and longitudinal contact stiffness. Theoretical investigation of normal and longitudinal interaction individually and both, have been presented as normal and longitudinal sensitivity. Also using the sensitivity equations the effects of material property and geometrical parameters can be specified.

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1. Introduction

The vibration of the tiny oscillators can be measured by tapping with an atomic force microscope (AFM). An AFM uses a tiny probe that moves slowly just above a surface. Electrostatic attraction or repulsion between the atoms in the tip of the probe and those in the surface causes the probe to move up and down, creating an image of the surface so detailed that individual atoms show up as bumps. Alternatively, the AFM can be used in tapping mode, literally bouncing off the surface. To measure the vibration of a Nano-Mechanical oscillator, the AFM probe moves along the length of the oscillating rod. The result is a complex bouncing interaction between the probe and the oscillator imagine shaking one end of a spring and watching the vibrations at the other end from which the frequency of vibration of the oscillator can be determined mathematically. To obtain atomic resolution, the AFM cantilever should not be too soft, and at the same time, it should have a high resonant frequency, in order to minimize sensitivity to vibrational noise from the building and to have a large imaging bandwidth. Furthermore, the resonant frequency of the cantilever can influence the imaging rate in the operating process. More information exists in the literature [1-5]. Dynamic responses of the AFM cantilever have been investigated by [6-8]. Some of researchers [9–14] have been studied the vibration response of an AFM cantilever for convenience without considering the interactive damping but Turner et al. [15] and Rabe et al. [16], have been shown the effect of damping on the vibration response of an AFM cantilever is very important. Chang et al. [17] have been investigated effect of interactive damping on sensitivity of vibration modes of rectangular AFM cantilevers. They found that the sensitivity of flexural mode 1 clearly decreases with increasing normal interactive damping coefficient and the higher damping coefficient can influence the larger range of β_n value. They also presented two equations for normal and torsional sensitivity. Effect of

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tip length and normal and lateral contact stiffness on the flexural vibration responses of atomic force microscope cantilevers have been investigated by Wu et al. [18]. They didn't consider the damping effects in their study. More recently Chang et al.[19] studied the sensitivity of the first four flexural modes of an AFM cantilever with a sidewall probe. Coupled lateral bending–torsional vibration sensitivity of atomic force microscope cantilever have been studied by Haw-Long Lee and Win-Jin Chang [20]. Their results showed that each mode has a different resonant frequency to variations in contact stiffness and each frequency increased until it eventually reached a constant value at very high contact stiffness. The normal and lateral interactive forces between the cantilever tip and the sample surface can be modeled by a set combination of a damper spring parallel to a dashpot in the normal direction and a similar combination in the lateral direction. Many researchers have studied the vibration response of an AFM cantilever, but there is no any investigation to consider the effects of normal and longitudinal contact stiffness both, by a set combination of a damper spring parallel simulation. At the current study analytical solution of interactive damping and normal and longitudinal contact stiffness on sensitivity of vibration modes of rectangular AFM cantilevers is presented.

2. Analysis

The schematic of the problem is shown in Fig.1. The cantilever has a length L, thickness b, width a, and tip length h. The tip interacts with the sample by a linear spring k_n and dashpots C_n for normal interaction and for longitudinal K_l and C_l for longitudinal interaction. It is assumed that the atomic force microscope cantilever here is a rectangular elastic beam and the dashpots are assumed to create a linear viscous type of damping.

2.1. Flexural vibration

When the AFM cantilever tip interacts with the sample by a normal spring K_n, normal dashpot C_n, and longitudinal spring k_l, longitudinal dashpots C_l the cantilever will vibrate flexural. The linear differential equation of motion for the free vibration of the cantilever beam is [21]

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,y)}{\partial t^2}=0 \tag{1}$$

Where E is the modulus of elasticity, I is the area moment of inertia, ρ is the volume density and there are related to material property and A is the uniform cross-sectional area of the cantilever.

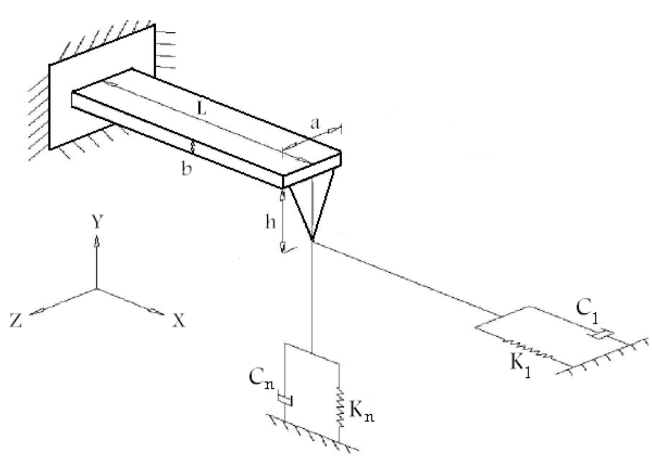

Fig. 1. Schematic of a rectangular AFM cantilever in contact with a sample.

The corresponding boundary conditions are

$$Y(0,t) = 0 \tag{2}$$

$$\frac{\partial y(0,t)}{\partial x} = 0 \tag{3}$$

$$EI\frac{\partial^2 y(L,t)}{\partial x^2} = -k_l h^2 \frac{\partial y(L,t)}{\partial x} - c_l h^2 \frac{\partial}{\partial t}\left(\frac{\partial y(l,t)}{\partial x}\right) \tag{4}$$

$$EI\frac{\partial^3 y(L,t)}{\partial x^3} = k_n y(l,t) + c_n \frac{\partial y(L,t)}{\partial t} \tag{5}$$

The boundary condition of the probe at $x = 0$ is assumed fixed end; then the boundary conditions given by Eq. (2), (3) correspond to conditions of zero displacement and zero slope. The boundary conditions given by Eq. (4), (5) correspond to zero moment at x=L and the force is balanced between the beam and a combination of the linear tip-sample stiffness and dashpot.
A general solution of equations. (1) to (5) is

$$y(x,t) = (a_1 \cos kx + a_2 \sin kx + a_3 \cosh kx + a_4 \sinh kx) e^{iwt} \tag{6}$$

where a_j, j=1-4 , are constants determined from the boundary conditions, w is the angular frequency, k is the flexural wave number.

$$w = k^2 \sqrt{\frac{EI}{\rho A}} \tag{7}$$

By setting Eq. (6) into Eq. (1)

$$EIk^4 - \rho A w^2 = 0 \tag{8}$$

Frequency can be shown as function of wave number

$$f = \frac{\gamma^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \tag{9}$$

From the quations, (2), (3), (6) can be found:

$$a_1 + a_3 = 0 \quad , \quad a_2 + a_4 = 0 \tag{10}$$

From the equations (4), (6):

$$a_1[-k^2 EI(\cos kL+\cosh L)-(k_l+c_l wi)h^2 k(\sin kL+\sinh kL)]+$$
$$a_2[-k^2 EI(\sin kL+\sinh kL)+(k_l+c_l wi)h^2 k(\cos kL-\cosh kL)]=0 \quad (11)$$

From the equations (5), (6):

$$a_1\left[-EIk^3(\sinh kL-\sin kl)+(k_n+c_n wi)(\cosh kL-\cos kL)\right]+$$
$$a_2\left[-EIk^3(\cosh kL+\cos kL)+(k_n+c_n wi)(\sinh kL-\sin kL)\right]=0 \quad (12)$$

By introducing

$$X_1=-EIk^2(\cos kL+\cosh kL)-(k_l+c_l wi)h^2 k(\sin kL+\sinh kL)$$
$$X_2=-EIk^2(\sin kL+\sinh kL)+(k_l+c_l wi)h^2 k(\cos kL-\cosh kL)$$
$$Y_1=-EIk^3(\sinh kL-\sin kL)+(k_n+c_n wi)(\cosh kL-\cos kL) \quad (13)$$
$$Y_2=-EIk^3(\cos kL+\cosh kL)+(k_n+c_n wi)(\sinh kL-\sin kL)$$

And with

$$\begin{vmatrix} X_1 & X_2 \\ Y_1 & Y_2 \end{vmatrix}=0 \quad (14)$$

Then the characteristics equation can be found:

$$C(\gamma,k_n,k_l)=\sum_{j=1}^{j=8}\eta_j=0 \quad (15)$$

Now suppose that

$$\varepsilon_1=[E^2I^2k^5+(k_n+c_n wi)(k_l+c_l wi)kh^2]$$
$$\varepsilon_2=[2EIk^4h^2(k_l+c_l wi)]$$
$$\varepsilon_3=[-2EIk^4h^2(k_l+c_l wi)+2EIk^2(k_n+c_n wi)] \quad (16)$$
$$\varepsilon_4=[2EIk^4h^2(k_l+c_l wi)+2EIk^2(k_n+c_n wi)]$$
$$\varepsilon_5=[2(k_n+c_n wi)(k_l+c_l wi)h^2k-2E^2I^2k^5]$$

And by introducing

$$\eta_1=\varepsilon_1\sinh^2\gamma$$
$$\eta_2=-\varepsilon_1\sin^2\gamma$$
$$\eta_3=-\varepsilon_2\sin^2\gamma\cos^2\gamma \quad (17)$$
$$\eta_4=-\varepsilon_3\sinh\gamma\cos\gamma$$

$$\eta_5 = -\varepsilon_4 \cosh\gamma \sin\gamma$$
$$\eta_6 = -\varepsilon_1 \cos^2\gamma$$
$$\eta_7 = \varepsilon_5 \cosh\gamma \cos\gamma$$
$$\eta_8 = -\varepsilon_1 \cosh^2\gamma$$

where γ=K×L is the normalized longitudinal wave number, $\beta_l=(K_l\times L^3/EI)$ is the normal stiffness ratio, $\beta_n=(K_n\times L^3/EI)$ is the normal stiffness ratio between the normal contact stiffness and that of the cantilever.
If we negligible C_n, K_n

$$C_l(\gamma,\beta_l)=\gamma(1+\cos\gamma\cosh\gamma)+\frac{h^2}{L^2}\left(\frac{\beta_l+c_l i\gamma^2 L}{\sqrt{\rho AE}}\right)[\sin\gamma\cos\gamma+\sinh\gamma\cos\gamma+\sin\gamma\cosh\gamma]=0 \quad (18)$$

The longitudinal and normal sensitivity of the cantilever can be calculated from the frequency, which can be measured. The sensitivity of the mode of the cantilever changes significantly for small variations of stiffness as the cantilever crosses the sample. Differentiation of Eq. (18) with respect to β_l yields

$$\frac{d\gamma}{d\beta_l}=-\frac{\partial c_1/\partial\beta_l}{\partial c_1/\partial\gamma} \quad (19)$$

The relationship between longitudinal frequency f_l and contact stiffness β_l can be expressed as

$$\frac{\partial f_l}{\partial\beta_l}=\frac{\partial f_l}{\partial\gamma}\,\frac{\partial\gamma}{\partial\beta_l} \quad (20)$$

Then the following equation can be obtained

$$\begin{aligned}\frac{df_l}{d\beta_l}=&\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}[-\frac{2\gamma h^2}{L^2}(\sin\gamma\cos\gamma+\sinh\gamma\cos\gamma+\sin\gamma\cosh\gamma)]\\&\times\{(1+\cos\gamma\cosh\gamma)+\gamma(\cos\gamma\sinh\gamma-\sin\gamma\cosh\gamma)\\&+\frac{2\gamma c_l iL}{\sqrt{\rho AE}}\frac{h^2}{L^2}(\sin\gamma\cos\gamma+\sinh\gamma\cosh\gamma+\sin\gamma\cosh\gamma)\\&+\frac{h^2}{L^2}(\beta_l+\frac{c_l\gamma^2 iL}{\sqrt{\rho AE}})(\cos^2\gamma-\sin^2\gamma+2\cos\gamma\cosh\gamma)\}^{-1}\end{aligned} \quad (21)$$

Equation (18) and (21) are complex types due to the complex representation of damping. The absolute value of the complex quantity is used in the calculation. Eq. (21) can be expressed in normalized form as

$$S_l = \frac{df / d\beta_l}{\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}} \tag{22}$$

By Setting K_l and C_l and equal to zero with neglected K_n and C_n, From the Eq. (15), the characteristics equation can be found:

$$C_2(\gamma,\beta_n) = \gamma^3\,(1+\cos\gamma\cosh\gamma) - (\beta_n + \frac{c_n i\gamma^2 L}{\sqrt{\rho AEI}})(\sinh\gamma\cos\gamma - \sin\gamma\cosh\gamma) \tag{23}$$

The normal sensitivity of the cantilever can be calculated from the frequency, which can be measured. The sensitivity of the mode of the cantilever changes significantly for small variations of stiffness as the cantilever crosses the sample. Differentiation of Eq. (23) with respect to β_n yields

$$\frac{d\gamma}{d\beta_n} = -\frac{\partial c_2 / \partial\beta_n}{\partial c_2 / \partial\gamma} \tag{24}$$

The relationship between normal frequency f_n and contact stiffness β_l can be expressed as

$$\frac{\partial f_n}{\partial\beta_n} = \frac{\partial f_n}{\partial\gamma}\ \frac{\partial\gamma}{\partial\beta_n} \tag{25}$$

Then the following equation is obtained

$$\begin{aligned}\frac{df_n}{d\beta_n} &= \frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}[2\gamma(\sinh\gamma\cos\gamma - \sin\gamma\cosh\gamma)]\\ &\times\{3\gamma^2\,(1+\cos\gamma\cosh\gamma) + \gamma^3\,(\sinh\gamma\cos\gamma - \cosh\gamma\sin\gamma)\\ &+2(\beta_n + \frac{c_n\gamma^2 iL}{\sqrt{\rho AEI}})\sinh\gamma\sin\gamma - 2ic_n\gamma L\sqrt{\frac{1}{\rho AEI}}(\sinh\gamma\cos\gamma - \cosh\gamma\sin\gamma)\}^{-1}\end{aligned} \tag{26}$$

That equation (26) is same as presented equation, for this case in [17]. The absolute value of the complex quantity is used in the calculation. Eq. (26) can be expressed in normalized form as

$$S_n = \frac{df / d\beta_n}{\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}} \tag{27}$$

With considering normal and longitudinal effects both, characteristics equation can be found as

$$C_3\,(\gamma, \beta_n, \beta_l) = \sum_{j=1}^{j=8} \phi_j = 0 \tag{28}$$

Where

$$\begin{aligned}
\phi_1 &= [\gamma^5 L + (\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})\frac{h^2\gamma}{L}]\sinh^2\gamma\,. \\
\phi_2 &= [-\gamma^5 L - (\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})\frac{h^2\gamma}{L}]\sin^2\gamma\,. \\
\phi_3 &= [-\frac{2\gamma^4 h^2}{L}(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})]\sin\gamma\cos\gamma\,. \\
\phi_4 &= [-\frac{2\gamma^4 h^2}{L}(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}}) + 2\gamma^2 L(\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})]\sinh\gamma\cos\gamma\,. \\
\phi_5 &= [-\frac{2\gamma^4 h^2}{L}(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}}) - 2\gamma^2 L(\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})]\sin\gamma\cosh\gamma\,. \\
\phi_6 &= [-\gamma^5 L - (\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})\frac{h^2\gamma}{L}]\cos^2\gamma\,. \\
\phi_7 &= [-2\gamma^5 L + 2(\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})\frac{h^2\gamma}{L}]\cos\gamma\cosh\gamma \\
\phi_8 &= [-\gamma^5 L - (\beta_n + \frac{c_n iL\gamma^2}{\sqrt{\rho AEI}})(\beta_l + \frac{c_l iL\gamma^2}{\sqrt{\rho AEI}})\frac{h^2\gamma}{L}]\cosh^2\gamma
\end{aligned} \tag{29}$$

For determine normal and longitudinal sensitivity the set of following equation are obtained

$$\frac{d\gamma}{d\beta_l} = -\frac{\partial c_3 / \partial\beta_l}{\partial c_3 / \partial\gamma} \tag{30}$$

$$\frac{\partial f_l}{\partial \beta_l} = \frac{\partial f_l}{\partial \gamma}\ \frac{\partial \gamma}{\partial \beta_l} \tag{31}$$

$$\frac{d\gamma}{d\beta_n} = -\frac{\partial c_3 / \partial \beta_n}{\partial c_3 / \partial \gamma} \tag{32}$$

$$\frac{\partial f_n}{\partial \beta_n} = \frac{\partial f_n}{\partial \gamma}\ \frac{\partial \gamma}{\partial \beta_n} \tag{33}$$

By introducing new parameters S_l and S_n as function of β_l and β_n for the first the effects of normal and longitudinal both, is considered. In the previous works parameter S_l isn't introduced and S_n is function of β_l only.

$$S_l(\beta_l, \beta_n) = \frac{df_l / d\beta_l}{\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}} \tag{34}$$

$$S_n(\beta_n, \beta_l) = \frac{df_n / d\beta_n}{\frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}} \tag{35}$$

The sensitivity parameter is also a functional of material property such as the modulus of elasticity and the volume density and geometrical property such as the area moment of inertia and cross-sectional area of the cantilever. The above equation denotes also the effects of this property.

3. Conclusion

The interactive damping and normal and longitudinal contact stiffness on sensitivity of vibration modes of rectangular AFM cantilevers has been investigated. The effect of damping on the vibration response of an AFM cantilever is significant and cannot be disregarded. Theoretical investigation of normal and longitudinal interaction individually and both, have been presented. Using the sensitivity equations the effects of material property and geometrical parameters can be specified.

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