SYNTHESIS OF CONVENTIONAL PHENOMENOLOGICAL THEORY OF SUPERCONDUCTIVITY WITH MARGINAL FERMI LIQUID MODEL

T. C. CHIBUEZE[a*], R. CHAUDHURY[b]
[a]*Department of Physics and Astronomy, University of Nigeria, Nsukka, Enugu State, Nigeria.*
[b]*S.N Bose National Center for Basic Sciences Kolkata, India.*

In this work we have done phenomenology based model calculations for some of the thermodynamic properties of the strongly correlated superconductors of Cuprate type. The method involves the application of the theoretical result for electronic specific heat in the normal phase from Marginal Fermi Liquid theory to the Gorter-Casimir two fluid model to further derive electronic specific heat and the temperature dependence of the critical magnetic field corresponding to a type-I system, using the standard variational technique. Our results are in fairly good agreement with other theoretical results based on different approaches, as well as with the experimental results.

(Received March 17, 2016; Accepted May 16, 2016)

Keywords: Two fluid, Marginal Fermi Liquid, Specific heat capacity, Cuprates.

1. Introduction

High temperature superconductivity in Cuprates has taken the centre stage of modern condensed matter physics since its discovery in 1987 because of the unusual normal state properties of these materials combined with the very rich phase diagram, besides the superconducting transition temperatures in the range of 40K-164K. These systems exhibit deviations from the Fermi liquid (FL) phenomenology in large regime of stoichiometric compositions [1,2]. Moreover, the conventional microscopic theory is not always successful to explain the properties in the superconducting phase satisfactorily. On a phenomenological level, the behaviour below the optimal doping in the normal phase seems to display ‘marginal Fermi liquid' (MFL) behaviour in the normal phase[3].

One of the most important features observed in experiments in the normal phase of the cuprates is the linear temperature dependence of dc resistivity, which below the optimal doping persists in an enormous temperature range from a few kelvin to much above room temperature[4]. The studies of the electrodynamic properties in the superconducting phase provide a clear phenomenological scenario, reveal information regarding the pairing state, the energy gap and the electronic density of states[5] and thus provide important indications on the mechanism of high temperature superconductivity.

A phenomenological model describing the MFL behaviour of cuprates has been put forward by Varma and co-workers[6,7] but its microscopic origin remains highly controversial. To our knowledge, no microscopic theory has so far been able to provide a satisfactory explanation for the phenomenon of high temperature superconductivity and anomalous normal phase properties of cuprates despite tremendous efforts during the last three decades[8].

*Corresponding author: timothy.chibueze@unn.edu.ng

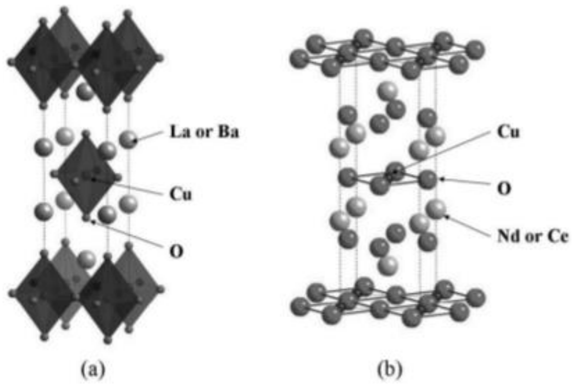

Fig. 1: Crystal structures of (a) $La_{2-x}Ba_xCuO_4$ and (b) $Nd_{2-x}Ce_xCuO_4$ superconductors.

Gorter and Casimir[9,10] in their two fluid (TF) model assumed the following formal analytical form for the free energy density of electron $F_s(x,T)$

$$F_s(x,T) = x^{\frac{1}{2}} f_n(T) + (1+x) f_s(T) \quad (1)$$

where $f_n(T)$ and $f_s(T)$ are the free energy density of the normal fluid and superfluid respectively and showed from the thermodynamic relation that the critical magnetic field $H_c(T)$ can be written in terms of the critical magnetic field at zero temperature H_o as

$$H_c(T) = H_o \left[1 - \left(\frac{T}{T_c}\right)^2\right] \quad (2)$$

Where T_c is the critical temperature.

Bardeen, Cooper and Schriefer (BCS)[11] proposed a microscopic Hamiltonian for a superconductor, which is based on the idea of Cooper pairing. Using this theory, they were able to successfully describe the interaction between electrons forming Cooper pair. The BCS theory has a parameter g defined as

$$g = N(0)V_{eff} \quad (3)$$

Where V_{eff} is the magnitude of the effective attractive interaction between the electrons forming a Cooper pair. From BCS equation in the weak coupling regime, one has the following equation for T_c,

$$T_c = 1.13\theta_c exp\left(-\frac{1}{g}\right) \quad (4)$$

Where θ_c is the temperature equivalent of the characteristic energy of the bosonic excitation mediating the pairing interaction. In the weak coupling regime, 0 < g < 0:25.
Hereafter we would assume equation (4) to be valid even when the pairing is mediated by high energy electronic boson.

In general, the unusual normal state properties of the high temperature superconducting copper oxide compounds indicate a scattering rate for the itinerant electrons, that is linear in frequency ω and linear in temperature T over a large region. This implies that these materials cannot satisfactorily be described by the conventional FL picture[8].

Varma et al[1,6] postulated that in the copper oxide system, there are charge and spin density fluctuations of the electronic system, which are significantly distinct from those in the conventional FL. These two excitations however have similar behaviour. These fluctuations lead to a new contribution to the polarisability of the electronic medium that would renormalize the electron through the self energy in accordance with the observed scattering rates[8,12].

Kuroda and Varma[13] calculated the specific heat of the MFL in the normal phase using a Fermi liquid-like formula in the presence of electron-boson coupling constant. This boson is taken to be the itinerant particle-hole pair (exciton) itself in the normal state. They obtained the electronic specific heat C_v of the MFL as

$$C_v = N(0)\left(3 + 2ln\frac{\theta_c}{T}\right)T \tag{5}$$

Where θ_c is the characteristic temperature corresponding to the energy of the excitonic boson in the MFL theory and assuming coupling coefficient $\lambda_+ = 1$.

2. Theory

We can extend the free energy of conduction electrons in a metal[14,15] and make use of the result of Kuroda and Varma[13] to arrive at the expression for the free energy density of the electrons in the normal phase of the MFL. We further make use of the TF model[9,10] and obtain the total electronic free energy density in the superconducting phase of the MFL from which we make use of the variational technique to obtain the electronic specific heat in the superconducting state C_v^S and the electronic specific heat in the normal state C_v^N as

$$C_v^S = N(0)\frac{T^3}{{T_c}^2}\frac{1}{\left(3+ln\frac{\theta_c}{T_c}\right)}\left[6\left(3 + ln\frac{\theta_c}{T}\right)^2 - 7\left(3 + ln\frac{\theta_c}{T}\right) + 1\right] \tag{6}$$

and

$$C_v^N = N(0)\left(3 + 2ln\frac{\theta_c}{T}\right)T \tag{7}$$

At the transition temperature T_c, the ratio of C_v^S to C_v^N, the difference between C_v^S and C_v^N called specific heat jump ΔC_v , the normalized specific heat jump R $(\frac{\Delta C_v}{C_v^N})$ and the normalized slope of the specific heat jump D were obtained and are shown in equations (8), (9), (10) and (11) respectively. The graphs are also shown in figures 3-5.

$$\frac{C_v^S}{C_v^N}|_{T=T_c} = \frac{1}{\left(3+2ln\frac{\theta_c}{T_c}\right)}\left[6\left(3 + ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+ln\frac{\theta_c}{T_c}\right)}\right] \tag{8}$$

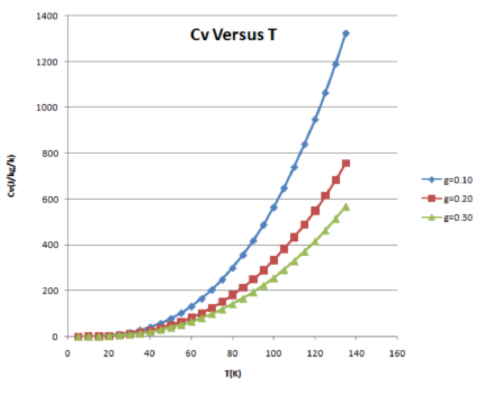

Fig. 2: Graph of specific heat in the superconducting phase C_v^S against temperature T.

$$\Delta C_v|_{T=T_c} = N(0)T_c \frac{1}{\left(3+ln\frac{\theta_C}{T_C}\right)}\left[6\left(3+ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+ln\frac{\theta_C}{T_C}\right)}\right] - N(0)\left(3+2ln\frac{\theta_c}{T_c}\right)T_c \qquad (9)$$

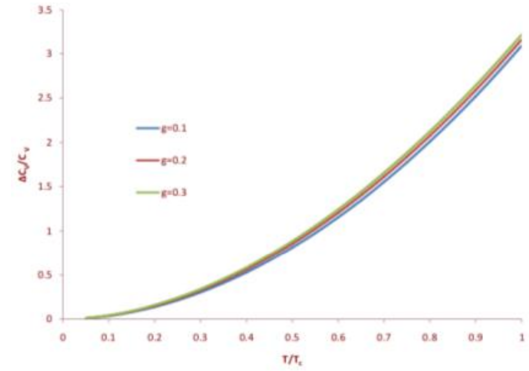

Fig. 3: Graph of normalized specific heat jump in the superconducting phase $\frac{\Delta C}{C_v^N}$ *against normalized temperature* $\frac{T}{T_c}$.

$$R = \frac{1}{\left(3+2ln\frac{\theta_C}{T_C}\right)}\left[6\left(3+ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+ln\frac{\theta_C}{T_C}\right)}\right] - 1 \qquad (10)$$

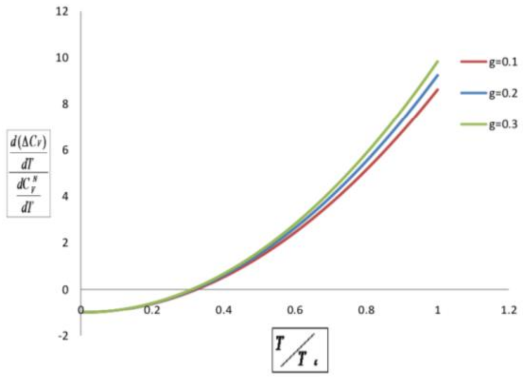

Fig. 4: Graph of normalized slope of the specific heat jump in the superconducting phase ΔC *against normalized temperature* $\frac{T}{T_c}$.

.
and

$$D = \frac{1}{\left(3+2ln\frac{\theta_C}{T_C}\right)}\left[18\left(3+ln\frac{\theta_c}{T_c}\right) - 33 + \frac{10}{\left(3+ln\frac{\theta_C}{T_C}\right)}\right] - 1 \qquad (11)$$

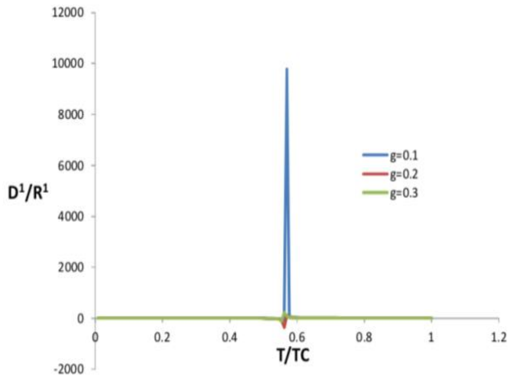

Fig. 5: Graph of ratio of the normalized slope of specific heat jump and normalized specific heat jump in the superconducting phase against normalized temperature $\frac{T}{T_c}$.

.

We also extended our expression for the free energy density of electrons in the normal phase of MFL and our C_v^N to obtain the temperature dependence of the critical magnetic field $H_c(T)$ for a superconductor arising from the MFL normal phase as

$$H_c(T) = H_o\left[1 - \left(\frac{\left(3+ln\frac{\theta_c}{T}\right)}{\left(3+ln\frac{\theta_c}{T_c}\right)}\right)\left(\frac{T}{T_c}\right)^2\right] \tag{12}$$

Where H_o is the critical magnetic field at T = 0 K.

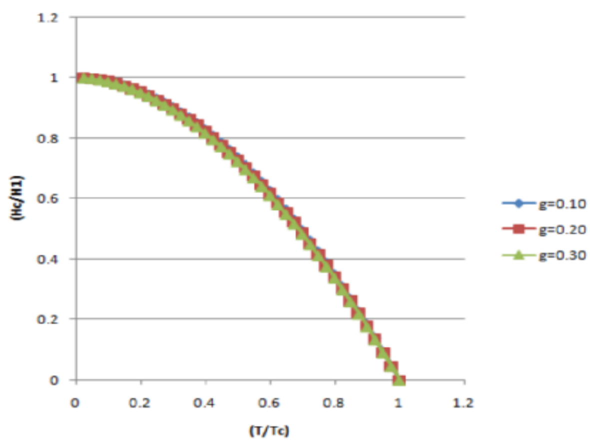

Fig. 6: Graph of $\frac{H_c}{H_o}$ against normalized temperature $\frac{T}{T_c}$.

Equation (12) is a departure from the conventional TF Model behaviour expected on the basis of the normal state modeled as a FL.

3. Discussion of Results

In our model, we have incorporated the normal phase properties described by MFL theory into the structure of Gorter-Casimir TF model. Specific heat measurements give information on the electron-boson coupling strength. The BCS theory[11] and its subsequent refinements based on the Eliashberg equations[16,17] show that high critical temperatures in superconductors are favoured by high values of the frequencies of the bosons mediating the pairing interaction and by the large electronic density of states at the Fermi level.

The quantity of interest is the difference between the electronic specific heats in the normal and superconducting phases. Our calculation shows that the normalized specific heat jump differs appreciably from 1.43, the value corresponding to the BCS weak coupling limit for a superconducting transition from the conventional FL phase[18] and from Khanna et al[19], values of 1.01 and 1.33 within the exotic pairing model for buckling mode and breathing mode respectively.

At low temperatures, the lattice contribution to the total specific heat is small and can be accurately subtracted to extract the purely electronic contribution. The normal phase specific heat can be obtained by applying a magnetic field of sufficient strength to cause the sample to become normal. From our result, the ratio of the normalized slope of specific heat jump and normalized specific heat jump in the superconducting phase $\frac{D}{R}$ at critical temperature T_c is 4.1197, 4.2616, and 4.4110 for g = 0.1, 0.2 and 0.3 respectively.

In the oxide superconductors, there are difficulties associated with these measure-ments. Because the superconducting critical temperatures of these oxide materials are relatively high, the lattice contribution to the total specific heat is quite large compared to the electronic contribution. An additional complication is that it is only possible to get normal state data close to critical temperature as the critical fields are quite large and are difficult to produce in the laboratory.

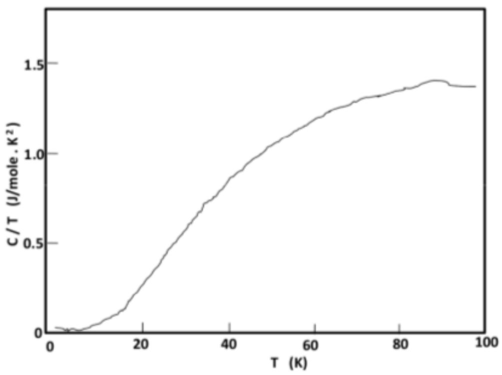

Fig. 7: Experimental result for the specific heat of Y $Ba_2Cu_3O_7$.

Fig. 7 represents the experimental results for specific heat corresponding to YBCO. Comparing with figure 2, observe that at low temperatures, there is an upturn in the specific heat rather than the expected exponential decay. However, there is still a linear term but there is no consensus yet on its origin. Analysis of the experimental data is usually done by assuming that the BCS relation $\frac{\Delta C}{\gamma T_c}$=1.43 holds. However it is pointed out by Beckman et al[20] that γ extracted by this analysis is not in good agreement with values from high T_c magnetization experiments and band structure calculations.

Loram et al[21] have used differential calorimetry on YBCO samples and report a normalized specific heat jump of 2.5. Wang et al[22] measured low temperature specific heat for single crystal cuprate (Y $Ba_2Cu_3O_{7-\delta}$) using thermal relaxation technique and obtained a value of about 4.0 for normalized specific heat jump. Philips et al[23] and Salas et al[5] have reported values of about 4.8 and 3.8 respectively.

From various observations, it would seem that there is a strong evidence for the specific heat jump to be large in the high T_c materials. This large value of the normalized specific heat jump is consistent with the result of ~3.02 in the model of synthesizing the Gorter-Casimir TF model with MFL theory as done in the BCS weak coupling regime in this work.

4. Conclusion

In this research, we have applied the results from the MFL theory to the Gorter-Casimir TF model and used these to calculate some thermodynamic properties like the specific heat jump and the temperature dependence of the critical magnetic field. The results of our calculations are closer to the experimental results obtained for Cuprates, than those from the phenomenological theory within the framework of ordinary FL assumptions, independently.

In this study, we have only modified the normal fluid part of the Gorter-Casimir TF model. A more accurate result can be obtained by modifying the super fluid part as well. One method of doing this is to use a scheme based on many body formalism which leads to the free energy of the full superconducting phase for a MFL superconductor. From this, one can in principle subtract the normal fluid free energy density and thereby extract the super fluid contribution corresponding to MFL.

Our methodology will be extended to a type-II superconducting system in future.

References

[1] C. M. Varma, Phys. Rev. B**55** 14554(1997).

[2] N. Plakida, High Temperature Cuprate Superconductors: Experiment, Theory and Applications Springer-Verlag, Berlin Heidelberg(2010).

[3] E. J. Nicol and J. P. Carbotte, Phys. Rev. B**43** 10210(1991).

[4] E.J Nicol, Pair-Breaking in Superconductivity. Ph. D Thesis, McMaster University, (1991).

[5] P. Salas, F.J Sevilla and M.A Solis, American. Phys. SocVol 50 No 1 10210(2013).

[6] P. B. Littlewood and C. M. Varma, Phys. Rev. B**46** 405(1992).

[7] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63** 1996(1989).

[8] C. Ranjan, Canadian Journal of Physics 73 (1995).

[9] C. J. Gorter and H. G. B. Casimir, Phys. Z 35 963(1934a).

[10] C. J. Gorter and H. G. B. Casimir, Z. Tech. Phys. 15 539(1934b).

[11] J. Bardeen, L. N. Cooper, and J. R. Schrie_er Phys. Rev. 108 1175(1957).

[12] D. I. Khomskii, Basic Aspects of the Quantum Theory of Solids: Order and Elementary Excitations, Cambridge University Press, Cambridge, (2010).

[13] Y.Kuroda and C.M. Varma, Phys. Rev. B**42** 8619(1990).

[14] N. W. Ashkroft and N. D. Mermin, Solid State Physics Harcourt College Publishers, New York, (1976).

[15] C. J. Poole Jr., H. A Farach, R. J. Creswick, R. Prozorov, Superconductivity Academic Press, Netherlands (2007).

[16] Y. M. Malozovsky, S. M. Bose, P. Longe, and J. D. Fan, Phys. Rev. B**48** 10504(1993).

[17] M. Schossmann, J. P. Carbotte and E. Schachinger Phys. Rev. B**46** 405(1992).

[18] J. B. Ketterson and S. N. Song, Superconductivity Cambridge University Press, Camdridge, (1999).

[19] K M Khanna, M. S. KarapKirui, T. W. Sakwa, P. K. Torongey, K. Y. Ayodo and S. Rotich Indian Journal of Pure and Applied Physics Vol. **45** 991-993(2007).

[20] O. Beckman, L. Lundgren, P. Nordblad, L. Sandlund, P. Svedlindh, T.Lundstrom, S. Rundqvist Physics Letters A**125** 425(2007).

[21] J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang, Phys. Rev. Lett. **71,** 1740(1993).

[22] Y. Wang, B. Revaz, A. Erb and A. Junod, Phys. Rev. B **63** 094508(2001).

[23] N.E. Phillips, R.A. Fisher, J.E. Gordon and S. Kim, Physica C: Superconductivity **162-164**, 1651(1989).