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FREE VIBRATION OF AN EMBEDDED CONICAL NANOTUBE WITH SURFACE EFFECT

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The vibration behavior of an embedded conical nanotube is studied with consideration of surface effect. The effects of the nanotube thickness, taper angle, surface parameter, and foundation constant on the natural frequency of the embedded conical nanotube with the clamped-clamped and clamped-free boundary conditions are analyzed. The Rayleigh–Ritz method with a higher order polynomial expansion technique is used in the calculation process. The method can also be used to solve the vibration problem of a uniform nanotube. For the case of a uniform nanotube, the present results are in excellent agreement with those obtained by the previous study using an analytical technique. According to the analysis, the natural frequency of the embedded conical nanotube with the clamped-free boundary condition obviously increases with an increase of the taper angle.

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1. Introduction

Conical nanostructures such as conical nanowires and nanotubes have been of significant interest to researchers because they have a wide range of potential applications in field emission and as atomic force microscope and near-field scanning optical microscope probes [1-4]. The nanostructures due to their unique features, which include a tapering geometry, provide more mechanical properties and thermal stability than their uniform ones. The bending stiffness of the nanostructures is dependent of cone angle and is expected to have higher than nanotubes and nanowires [5], and they also have better resistance to thermal-induced drift than nanowires [6].

A conical nanotube can be made from diverse materials such as Si, C, Au, and Al and is a new morphological manifestation of nanotube with a high aspect ratio, hollow geometry, and unique structure. The conical nanotube has many potential applications in nanobiological devices and nanomechanical systems. For example, Harrell et al. [7] developed a device, which consisted of a single conically shaped gold nanotube embedded within a polymeric membrane. This device allowed the ability to strongly rectify the ionic current flowing through it. Sexton et al. [8] utilized a conical nanotube sensor to detect protein analytes. Huang et al.[9] built the polypyrrole conical nanocontainers, which may have potential application in biological and controllable drug delivery involving extremely small amounts of active substances.

For nanoscale materials, due to high surface-to-volume ratio, surface effect becomes significant and cannot be neglected in the analysis of physical properties. Recently, many researchers have studied mechanical and vibration properties of the nanotubes and nanowires including the surface effect. For example, Lee and Chang [10] studied the surface effect on the vibration behaviors of conical nanowires and nanotubes. They found that the natural frequency of nanotubes increased when the surface effect was taken into account in the analysis. Assadi and Farshi [11] investigated the longitudinal and transverse wave propagation in embedded nanotubes with consideration of surface effect. Recently, Park [12] studied the surface effect on the critical

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buckling strains of silicon nanowires with cross sectional sizes ranging from 10 to 25 nm.

In this article, the vibration frequency of an embedded conical nanotube with clamped-clamped and clamped-free boundary conditions and surface effect is studied. In addition, the effects of conical angle, ratio of tube length to tube thickness, elastic foundation on the natural frequency are analyzed using the Rayleigh–Ritz method.

2. Analysis

A schematic diagram of a conical nanotube (or nanoshell) is depicted in Fig. 1. An actual nanotube was fabricated using a lithography technique as shown in Figs. (1a) and (1b) [13]. The inner radius R_i , and outer radius R_o of the conical nanotube are linearly varied along x axis and its length is L as shown in Figs. (1c) and (1d). The nanotube with the taper angle α has the inner radius R_{i0} of the larger edge located at the x =0, Young's modulus $E,$ and volume density ρ. Meanwhile, the inner and outer surfaces of the nanotube are represented by two bonded layers modeled as the inner and outer shells (S) include the effect of an additional surface elasticity E_s [11]. The surface effect in the analysis is considered as a uniform surface layer with infinitesimal thickness [14-15]. In addition, the conical nanotube is assumed to be embedded in an elastic foundation.

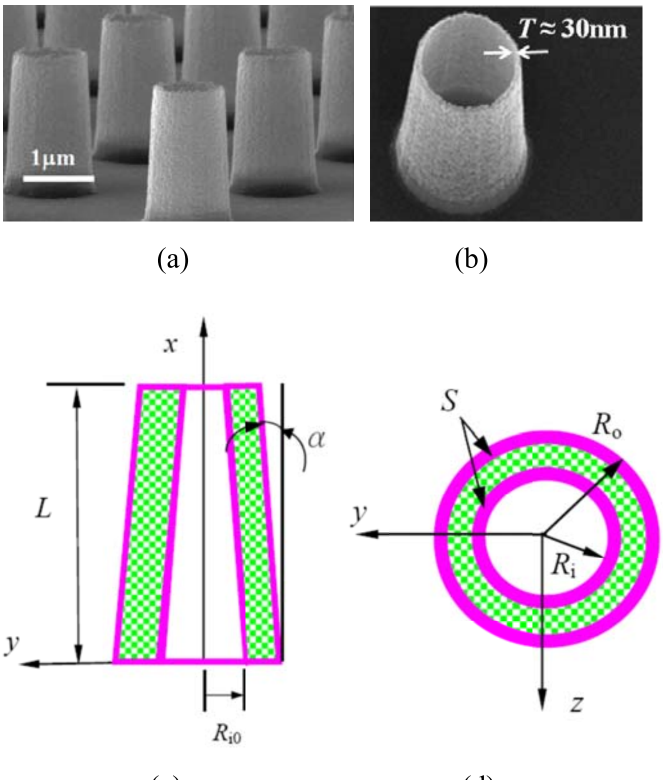


Fig . 1. SEM images of nanoshell array from the (a) Front view and (b) magnified top view [13]. (c) Schematic illustrate of a conical nanoshell. (d) The cross section of the nanoshell with shell-core-shell configuration.

By the Rayleigh–rod model, the governing equation for longitudinal vibrations of the rod can be expressed as[16]

$$\frac{\partial}{\partial x}[EA\frac{\partial u}{\partial x}]-\rho A\frac{\partial^2 u}{\partial t^2}+\rho v^2\frac{\partial}{\partial x}(J\frac{\partial^3 u}{\partial t^2 \partial x})=Ku\,, \tag{1}$$

where t is time, u is the longitudinal displacement; $J=\int_A r^2 dA=\frac{\pi}{2}(R_o^4-R_i^4)$ is the polar moment of inertia, ν is Poisson's ratio, it is used to take into account transverse vibrations proportional to the rod longitudinal deformation, and K is the elastic foundation constant in the longitudinal direction.

The boundary conditions for clamped-clamped are

$$u(0,t)=0 \text{ and } u(L,t)=0 \tag{2}$$

In addition the boundary conditions for clamped-free are

$$u(0,t)=0, \text{ and } \frac{\partial u(L,t)}{\partial x}=0. \tag{3}$$

In order to study the surface effect, the axial rigidity EA given should be modified as the effective axial rigidity $(EA)^*$ which is

$$(EA)^*=EA_c+E_sA_s=\pi E(R_o^2-R_i^2)+2\pi E_s(R_o+R_i), \tag{4}$$

where E_cA_c and E_sA_s denote the axial rigidity of the nanotube core and surface layers, respectively. We seek harmonic solution of the form as

$$u(x,t)=\theta(x)e^{j\omega t}. \tag{5}$$

where ω is the angular frequency, $\theta(x)$ is the longitudinal vibration mode.

To simplify the manipulation, the following dimensionless variables are introduced:

$$\xi=\frac{x}{L},\quad \psi=\frac{\theta}{L},\quad e=\frac{E_s}{EL},\quad k=\frac{K}{E_c},\quad \mu=\frac{\omega}{\sqrt{E/\rho L^2}}, \tag{6}$$

where e, k, and μ are the dimensionless surface parameter, foundation constant, and natural frequency, respectively.

Meanwhile, the dimensionless inner and outer radii of the conical nanotube can be expressed as

$$r_i(\xi)=\frac{R_{i0}}{L}(1-\xi\tan\alpha),\qquad r_o(\xi)=r_i+\frac{t_c}{L}, \tag{7}$$

where t_c represents the thickness of the conical nanotube, which is equal to R_o-R_i.

Using Eqs. (5)-(7), the governing equation and boundary conditions can be rewritten in the following dimensionless forms:

$$\frac{d}{d\xi}\{[\pi(r_o^2-r_i^2)+2\pi e(r_o+r_i)]\frac{d\psi}{d\xi}\}-k\psi=-\mu^2[\pi(r_o^2-r_i^2)\psi-\nu^2\frac{d}{d\xi}(\frac{\pi}{2}(r_o^4-r_i^4)\frac{d\psi}{d\xi})], \tag{8}$$

$$\psi(0)=0 \text{ and } \psi(1)=0, \tag{9}$$

and

$$\psi(0)=0 \text{ and } \frac{d\psi(1)}{d\xi}=0 \tag{10}$$

Since the parameters r_o and r_i are dependent on the ξ, the analytical solution to the above problem cannot be obtained. Here, the Rayleigh-Ritz method with a ten-term polynomial expansion technique is used to determine the frequency. We set

$$\psi(\xi)=\sum_{i=1}^{n}\beta_i\gamma_i(\xi), \tag{11}$$

where β_i is any constant, and $\gamma_i(\xi)$ is the admissible function which required to satisfy the geometric boundary conditions, but need not satisfy the natural boundary conditions. Here, the admissible function is assumed to be $\gamma_i(\xi)=\xi(1-\xi)^i$ for the clamped-clamped conical nanotube and $\gamma_i(\xi)=\xi^i$ for the clample-free one.

Then, substituting Eq. (11) into Eq.(8) and applying the Rayleigh quotient, the following eigenvalue problem can be obtained:

$$\beta H=\mu^2 M\beta , \tag{12}$$

where μ is the eigenvalue and β is the corresponding eigenvector, and

$$H_{ij}=\int_0^1\{\pi[(r_o^2-r_i^2)+2e(r_o+r_i)]\frac{d\gamma_i}{d\xi}\frac{d\gamma_j}{d\xi}+k\gamma_i\gamma_j\}d\xi , \tag{13}$$

$$M_{ij}=\int_0^1\pi[(r_o^2-r_i^2)\gamma_i\gamma_j+v^2(\frac{1}{2}(r_o^4-r_i^4)\frac{d\gamma_i}{d\xi})\frac{d\gamma_j}{d\xi}]d\xi . \tag{14}$$

By solving Eq.(12) and using the dimensionless variables given by Eq. (11), the natural frequency of longitudinal vibration of the conical nanotube can be obtained as

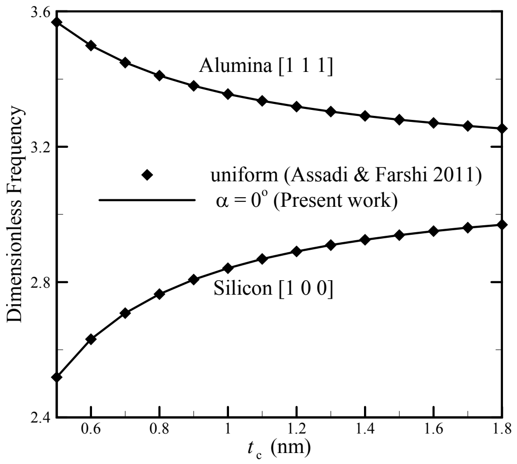
$$\omega=\mu\sqrt{E/\rho L^2} . \tag{15}$$

In case of setting $\alpha=0$ as given in Eq. (7), it implies that the conical nanotube becomes a uniform one. And then the natural frequency of longitudinal vibration of the uniform nanotube can be obtained.

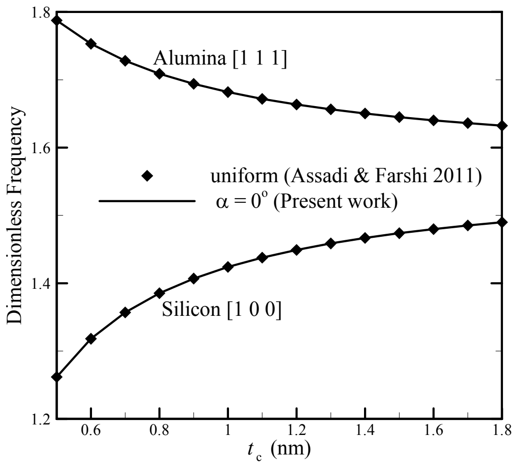
3. Results and discussion

In this article, the effects of the nanotube thickness, R_o-R_i (i.e., t_c), taper angle, surface parameter, E_s/EL, and the ratio of the elastic modulus of the nanotube to foundation constant, *K/E*, on the vibration frequency of a conical nanotube are investigated. The Rayleigh–Ritz method with a ten-term polynomial expansion technique is adopted in the calculation process. In order to verify the accuracy of the proposed approach, the present results are compared with those obtained using the analytical method by Assadi and Farshi for the uniform nanotube [11]. We use the same material properties in the previous model [11] for a silicon [100] nanotube as $\mathrm{E}=130$ GPa, $\nu=$ 0.28, $\rho=2330\ \mathrm{kg/m^3}$, and $\mathrm{E_s}=-11.5\ \mathrm{N/m}$, and for an anodic alumina [111] nanotube as $\mathrm{E}=70$ GPa, $\nu=0.3$, $\rho=2700\ \mathrm{kg/m^3}$, and $\mathrm{E_s}=5.1882\ \mathrm{N/m}$. It is noted that the silicon

[100] nanotube has a negative surface parameter, but that is positive for the anodic alumina [111] nanotube. The natural frequency of a material with a positive surface parameter is higher than that predicted using the conventional theory without surface effects. Figs. 2 (a) and 2 (b) depict the comparison of the present results for the case of $\alpha = 0$ with the previous study for the silicon and alumina uniform nanotubes on the clamped-clamped and clamped-free boundary conditions without considering the foundation medium (i.e., k = 0), respectively. With an increase of t_c, the frequency of the silicon nanotube appears to be opposite to that seen in the alumina nanotubes for two both boundary conditions due to the different surface effect. It can be seen that the comparison of the results with different methods are almost identical for different materials and boundary conditions.



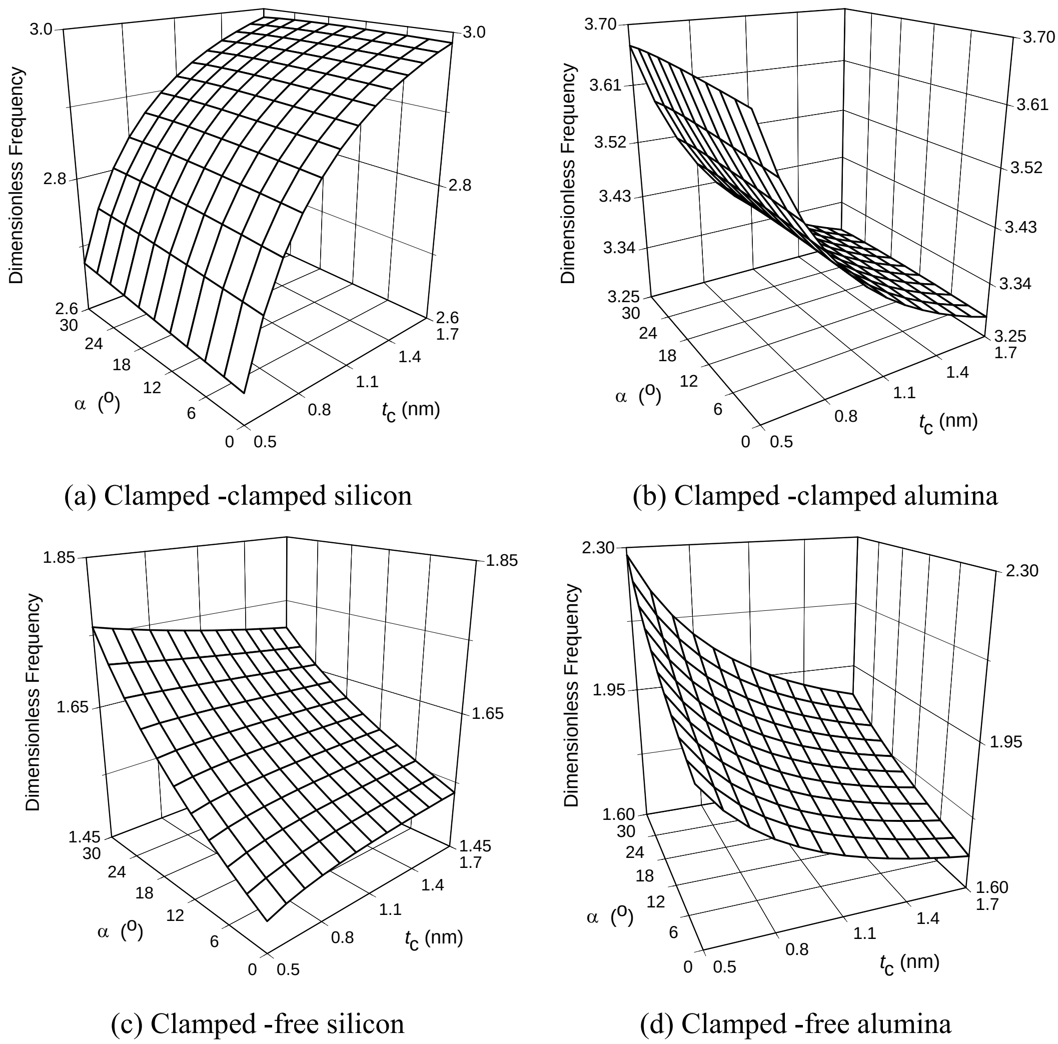
(a) Clamped –clamped



(b) Clamped -free

Fig. 2. the comparison of dimensionless fundamental frequency of the (a) Clamped -clamped and (b) Clamped -free uniform nanotubes of different materials with k = 0, L= 30 nm, and R_{i0} *= 2 nm.*

Fig. 3 shows the dimension fundamental frequency of the embedded conical nanotube as functions of α and t_c with $K/E = 0.5\times10^{-2}$ and L = 30 nm. For a uniform nanotube (i.e., $\alpha = 0$), the frequency of the conical silicon nanotube with the clamped-clamped and clampled-free boundary conditions increases when the value of t_c increases. But the situation is reverse for the conical alumina nanotube. This is because their surface effect are different as discussed in Fig.(2a) and (2b). The surface effect gradually decreases with increasing the value of t_c. In addition, with increasing the value of α, the frequency of the conical nanotune almost linearly increases. This is because the mass of the conical nanotube decreases and then the frequency increases. Therefore, it can be seen in Fig. 3(c) that the frequency of the conical clamped-free silicon nanotube with a larger taper angle decreases when the value of t_c increases.



(a) Clamped -clamped silicon

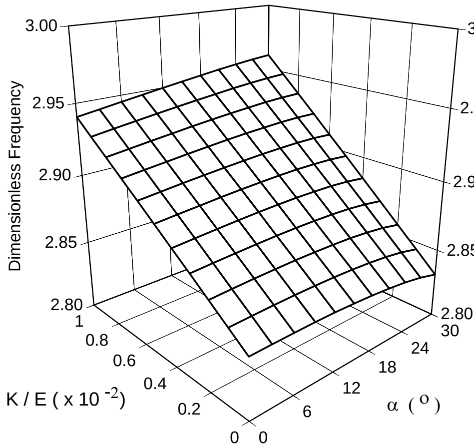
(b) Clamped -clamped alumina

(c) Clamped -free silicon

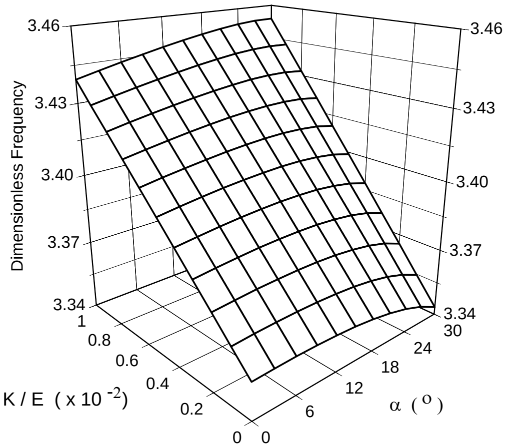
(d) Clamped -free alumina

Fig. 3. Dimension fundamental frequency of the (a) clamped -clamped silicon, (b) clamped -clamped alumina, (c) clamped -free silicon, and (d) clamped -free alumina conical nanotube as functions of α *and* t_c *with* $K/E = 0.5\times10^{-2}$ *and L = 30 nm.*

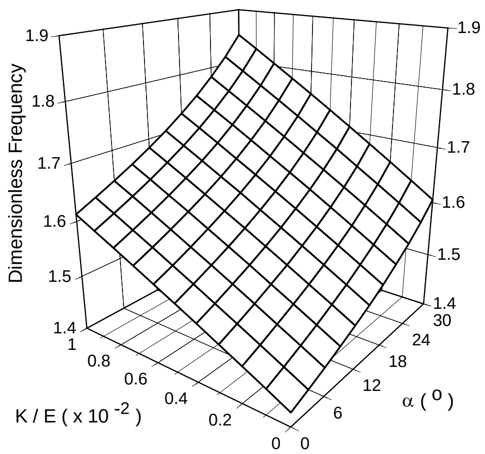
Fig. 4 depicts the dimension fundamental frequency of the embedded conical nanotube with L = 30 nm and $t_c = 1.0nm$ as a functions of K/E and α. When the foundation effect is not considered (i.e., K/E=0), the dimensional frequency is slightly decrease with increasing taper angle for the clamped-clamped conical nanotube, but it obviously increase for the clamped-free conical nanotube. This trend is the same with the previous study [17]. If the foundation effect is included in the analysis, it makes the nanotube become stiffer and then the frequency becomes high. In addition, the mass of the conical nanotube decreases with increasing the taper angle, and then that leads to the frequency increases. Therefore, it can be found that the frequency of the conical nanotube obviously increases with increasing the value of K/E, especially at a larger taper angle. Furthermore, the frequency of the conical alumina nanotube is higher that of the conical silicone nanotube for the same conditions.



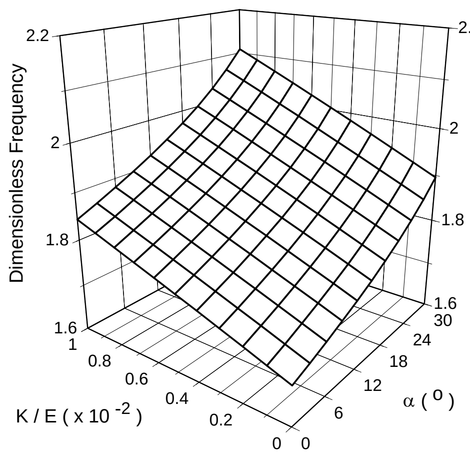
(a) Clamped -clamped silicon



(b) Clamped -clamped alumina

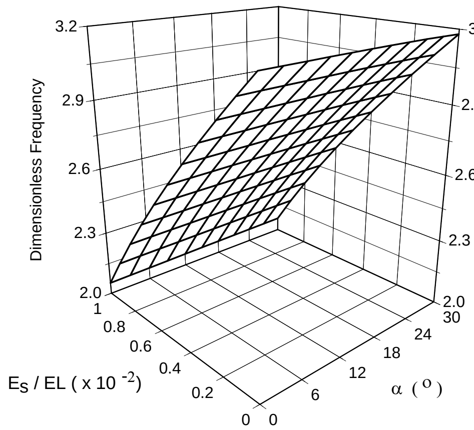


(c) Clamped -free silicon

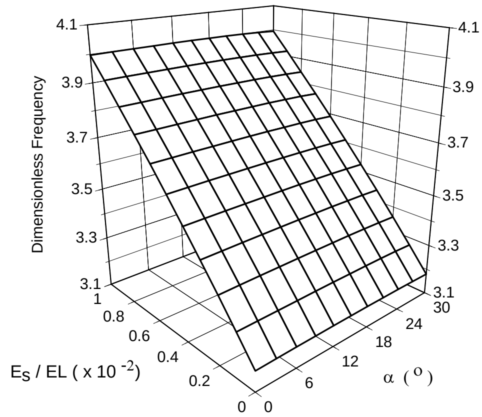


(d) Clamped -free alumina

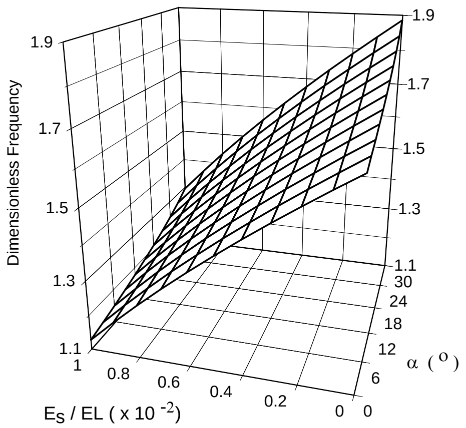
Fig. 4. Dimension fundamental frequency of the (a) clamped -clamped silicon, (b) clamped -clamped alumina, (c) clamped -free silicon, and (d) clamped -free alumina conical nanotube with L = 30 nm and $t_c = 1.0nm$ *as a functions of* K/E *and* α.



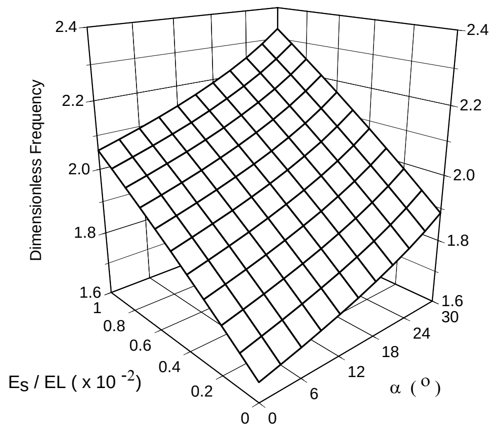
(a) Clamped -clamped silicon



(b) Clamped -clamped alumina



(c) Clamped -free silicon



(d) Clamped -free alumina

Fig. 5. Dimension fundamental frequency of the (a) clamped -clamped silicon, (b) clamped -clamped alumina, (c) clamped -free silicon, and (d) clamped -free alumina conical nanotube as functions of α *and* E_s / EL *with* $K / E = 0.5 \times 10^{-2}$*, L = 30 nm and* $t_c = 1.0nm$.

Fig. 5 illustrates the dimension fundamental frequency of the embedded conical nanotube as functions of α and E_s / EL with L = 30 nm, $t_c = 1.0nm$, and $K / E = 0.5 \times 10^{-2}$. The surface effect on the natural frequency of the conical nanotube is significant. A material with a negative surface parameter appears to have a lower frequency than that predicted by conventional theory without surface effects. Therefore, the frequency of the conical silicon nanotube decreases with increasing the value of E_s / EL. The trend is reverse for the conical alumina nanotube with a positive surface parameter.

4. Conclusions

The effects of the ratio of the nanotube elastic modulus to foundation constant, nanotube thickness, taper angle, and surface parameter on the natural frequency of a conical nanotube on the clamped-clamped and clamped-free boundary conditions are analyzed. The results showed that the frequency of the conical nanotube almost linearly increased with an increase of the taper angle. The increase situation is more obvious for the clamped-free nanotube. The frequency of the conical nanotube apparently increased with increasing the ratio of the elastic modulus of the nanotube to foundation constant, especially at a larger taper angle. In addition, with increasing the surface parameter, the natural frequency of the conical silicon nanotube obviously decreased. But the frequency of the conical alumina nanotube apparently increased.

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