Digest Journal of Nanomaterials and Biostructures Vol. 15, No. 2, April - June 2020, p. 435 - 446

ON SUSPENSION STABILITY OF NANOPARTICLES TO ENHANCE THERMAL PERFORMANCE OF MHD NANOFLUID FLOW OVER A STRETCHABLE ROTATING DISK WITH BIOCONVECTION EFFECTS

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Heat and flow characteristics for bioconvection of nanofluid due to a radially stretching and rotating disk are examined. The nanofluid bioconvection is caused by the combined effects of magnetic field and buoyancy force on the interaction of motile gyrotactic microorganisms and nanoparticles in a dilute base fluid. The doping of suspended gyrotactic microorganisms and nanoparticles produces innovative heat transfer enhancement with provision of higher thermal conductivity through stability of nano particles. The motile microorganisms are self-propelled and they can actively swim in the fluid in response to such stimuli as gravity, light or chemical attraction. The computational results for physical quantities of interest due to influential thermophysical parameters and bioconvection parameters have been evaluated by employing bvp4c solver in Matlab. It is observed that the cooling rate becomes faster when radial stretching, Brownian diffusion and thermophoretic diffusion are increased.

(Received December 10, 2019; Accepted May 5, 2020)

Keywords: MHD flow, Nanoparticles, Thermal performance, Heat Transfer, Rotating / Stretching surface, Bioconvection

1. Introduction

Effective cooling techniques are greatly needed for cooling any sort of high-energy device. Nanometer sized structures are being produced in the form of functional nanoscopic structures having utilization in different environments. Thermal conductivity can be increased by adding metals to the base fluids. The resultant fluids are termed as nanofluids. This classical idea was first introduced by Choi [1]. Theoretical and experimental studies of the performance of different nanoparticles such as Al_2O_3, Cu, CuO, TiO_2 and Ag have been conducted by Akbarzadeh et al. [2], Oztop and Abu-Nada [3], Mebrouk et al. [4]. Hussanan et al. [5] analyzed MHD flow of a nanofluid over an accelerating infinite vertical plate though a porous medium in the presence of thermal radiation and used water as conventional base fluid containing three different types of nanoparticles, namely copper (Cu), alumina (Al_2O_3), titanium dioxide (TiO_2). Aaiza et al. [6] investigated the radiative heat transfer in mixed convection MHD flow of a different shapes of Al_2O_3 in EG-based nanofluid in a channel filled with saturated porous medium. Asma et al. [7] obtained exact solutions for free convection flow of nanofluids with ramped wall temperature by taking five different types of spherical shaped nanoparticles.

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The density gradient resulting from collective swimming of motile micro-organisms and the macroscopic fluid motion is defined as Bioconvection [8]-[10]. The micro-organisms under consideration are orientated by a balance between a gravitational torque, due to them being bottom heavy, and viscous torque arising from local fluid velocity gradients. The displacement between the center of buoyancy and mass, Oxygen concentration gradient and negative gravity and are common stimulators of these microorganisms [8]. The history of bioconvection is not so long, Kuznetsov [11], firstly discussed studies about nanofluid bioconvection in 2010. Khan and Makinde [12] investigated the MHD boundary layer flow of a water-based nanofluid containing motile gyrotactic micro-organisms along a linearly stretching sheet. Sharma and Kumar [13] studied the linear stability of bioconvection in a dilute suspension of gyrotactic microorganisms in horizontal shallow fluid layer cooling from below and saturated by a porous medium.

Recently, Moli et al. [14] utilized a randomly swimming model to investigate the stability of bioconvection in a horizontal suspension layer of motile gyrotactic micro-organisms. The micro-organisms under consideration are orientated by a balance between a gravitational torque, due to this being bottom heavy, and viscous torque arising from local fluid velocity gradients. Zohra et al. [15] studied natural convective anisotropic slip boundary layer flows from a rotating vertical cone embedded in ethylene glycol bionanofluid with four different nanoparticles namely Copper (Cu), Alumina (Al_2O_3), Copper Oxide (CuO), Titanium Oxide (TiO_2). Saini and Sharma [16] discussed the bio-thermal convection in a suspension containing gravitactic microorganisms saturated by a fluid is investigated within the framework of linear and nonlinear stability theory. Soida et al. [17] investigated steady magnetohydrodynamic (MHD) flow past a radially stretching or shrinking disk.

The current investigation embraced the enhanced thermal transportation with bioconvection effects for MHD nanofluid flow due to a radially stretching and rotating disk. The dilute suspension of nano-sized metallic particles adheres to incremented thermal conductivity of the base fluid. In order to avoid any chance of sedimentation of metallic particles, we extended the work in [17] with bioconvection due to self- motile gravitactic microorganism induced in the nanofluids. The physical nature of fluid flow, fluid temperature, volume fraction of nano particles and density function of motile microorganisms is revealed through variation of controlling parameters that emerged in the formulation of the problem. Numerical solution is obtained by implementing bvp4c function of Matlab software (Matlab_R2018b, MathWorks, Singapore).The outcomes of present work are validated by manifestation of their good agreement with those in existing literature.

2. Mathematical formulation

The behavior of a water-based nanofluid containing motile gyrotactic micro-organisms passing a radially stretching-rotating sheet is studied numerically. The nanoparticles do not have any influence on both the swimming velocity and swimming direction of microorganisms. It is taken for granted that stability of resulting nanofluid suspension is maintained and it is dilute (nanoparticles concentration less than 1%) to uphold the gyrotactic microorganisms bioconvection in optimum manner. Since high viscosity of base fluid pose negative influence on bioconvection, it is imperative to dilute the suspension containing nanoparticles. The nanoparticles suspension is stable and dilute such that there is no agglomeration and accumulation of nanoparticles. It should be noted that increasing concentration of nanoparticles leads to the instability. It is supposed that the nanoparticles have no effect on the direction and velocity of microorganism's swimming.

The governing equations for the physical problem are presented below as referred to Mushtaq and Mustafa [18]:

$$\nabla . V = 0 \tag{1}$$

$$\begin{aligned}\rho_f (V.\nabla) V = -\nabla \rho + \mu_f \nabla^2 V + J \times \beta + (1 + C_\infty) \rho_f \beta g (T - T_\infty) \\ + \begin{bmatrix} -(\rho_p - \rho_f) g (C - C_\infty) \\ -(n - n_\infty) g \gamma (\rho_m - \rho_f) \end{bmatrix},\end{aligned} \tag{2}$$

$$\begin{aligned}(V.\nabla) T = \alpha_f \nabla^2 T + \tau \left(D_B (\nabla C . \nabla T) + \frac{D_T}{T_\infty} (\nabla T)^2 \right) \\ - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y}\end{aligned} \tag{3}$$

$$(V.\nabla) C = D_B \nabla^2 C + \frac{D_T}{T_\infty} \nabla^2 T \tag{4}$$

$$V . J_1 = 0 \tag{5}$$

In above equations $V(u,0,w)$ is the velocity vector, J is the current density, B is the magnetic flux vector, ρ_f, ρ_p, ρ_m are the density of fluid, nanoparticles and microorganisms respectively. T is the temperature, k is the thermal conductivity, β is coefficient of thermal expansion, g is acceleration due to gravity, $(\rho c)_f$ capacitance of the base fluid, $(\rho c)_p$ is the heat capacitance of the nano particles and τ is their ratio, C is the concentration, $\alpha_f = k/(\rho c)_f$ is thermal diffusivity of nanofluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, $(\rho c)_f$ the liquid heat capacity, $(\rho c)_p$ the effective nanoparticles heat capacity, q_r the radiative heat flux, J_1 the flux of microorganisms due to macroscopic convection of fluid, self-propelled swimming and diffusion of microorganisms defined by

$$J_1 = nV + n\hat{V} - D_n \nabla n \tag{6}$$

where n shows the density motile of microorganisms, $\hat{V} = (b_1 W_c / \Delta C) \nabla C$ the velocity related to cell swimming, b_1 the chemotaxis constant, W_c is maximum cell swimming speed and D_n the microorganisms diffusion coefficient.

The appropriate boundary conditions are:

$$\begin{aligned}& u = sr,\ v = r\Omega,\ -k_f \frac{\partial T}{\partial z} = h_f (T_f - T),\ D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0,\ n \to n_0 \text{ at } z = 0 \\ & u \to 0,\ n \to n_\infty,\ T \to T_\infty,\ C \to C_\infty \text{ as } z \to \infty.\end{aligned} \tag{7}$$

Using the Rosseland approximation, the radiative heat flux q_r

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial z}, \tag{8}$$

with the assumption that is expanded in Taylor series about and neglecting higher order terms to get

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{9}$$

By using the (8) and (9) , we obtain the expression as follows

$$\frac{\partial q_r}{\partial z} = -\frac{16\sigma^* T_\infty^3}{3K^*}\frac{\partial^2 T}{\partial z^2} \tag{10}$$

Then equation (3) can be written as

$$\begin{aligned} u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha_f\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{(\rho c)_p}{(\rho c)_f}(D_B\left(\frac{\partial T}{\partial z}\frac{\partial C}{\partial z} + \frac{\partial T}{\partial r}\frac{\partial C}{\partial r}\right) \\ + \frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial z}\right)^2 + \left(\frac{\partial T}{\partial r}\right)^2\right)) + \frac{16\sigma^* T_\infty^3}{3K^*}\frac{\partial^2 T}{\partial z^2}. \end{aligned} \tag{11}$$

We employed the self-similar transformation for u, w, T, C, n by $F^{/}, G, \theta, \phi, \chi$ respectively in the term of the dimensional vertical distance $\zeta = z\left(\frac{2\Omega}{\nu_f}\right)^{\frac{1}{2}}$ and obtained as follows:

$$F''' + FF'' - \frac{1}{2}G^2 - MF' - KF' - \lambda_1(\theta - Nr\phi - Nc\chi) = 0 \tag{12}$$

$$G'' + FG' - GF' - K_1 G - \lambda_1 G = 0 \tag{13}$$

$$\frac{1}{P_r}(1+R)\theta'' + F\theta' + Nb\theta'\phi' + Nt(\theta')^2 = 0 \tag{14}$$

$$\phi'' + ScF\phi' + \frac{Nt}{Nb}\phi'' = 0 \tag{15}$$

$$\chi'' + Lbf\chi' - Pe(\phi''(\chi + \delta_1) + \chi'\phi') = 0 \tag{16}$$

$$\begin{aligned} &F = 0,\ F' = c,\ G = 1,\ \theta' = -Bi(1-\theta),\ \phi' + \frac{Nt}{Nb}\theta' = 0,\ \chi(0) = 1 \text{ at } \zeta = 0, \\ &F' \to 0,\ G \to 0,\ \theta \to 0,\ \phi \to 0,\ \chi \to 0 \text{ as } \zeta \to 0 \end{aligned} \tag{17}$$

where the dimensionless constants are given blow:

$Sc = \nu_f / D_B$ is Schmidt number, $Lb = \alpha_f / D_n$ is bioconvection Lewis number, $Pe = bW_c / \nu_f$ is bioconvection Peclet number, R is radiation parameter, $P_r = \frac{\nu_f}{\alpha_f}$, $P_{reff} = \frac{P_r}{1+R}$ is Prandtl number, λ_1 is buoyancy parameter, Brownian motion parameter is $Nb = \frac{\tau D_B C_\infty}{\nu_f}$, Thermopherosis parameter is $Nt = \frac{\tau D_T (T_f - T_\infty)}{T_\infty \nu_f}$, $Bi = \frac{h_f}{k}\left(\frac{\nu_f}{2\Omega}\right)^{\frac{1}{2}}$, N_c is Raleigh number, δ_1 is microorganism difference parameter, N_r is buoyancy ratio parameter and c is stretching parameter.

3. Results and discussion

The set of simultaneous non-linear ordinary differential equations viz Eq. (12) to (17) are difficult to yield any closed form solution. A numerical treatment bvp4c in Matlab environment is employed to obtain numerical solution to reveal the physical nature of the problem under the influence of emerging parameters. Some representative results for velocity, temperature, nano particle concentration and microorganism density function are computed.

Fig. 1 presents the influence of stretching-strength parameter c on radial velocity $F'(\varsigma)$ and tangential velocity $G(\varsigma)$. Physically, the parameter c stands for the ratio of stretch rate to the swirl rate and it can attain the range $0 \le c \le \infty$. When $c = 0$, there is no radial stretching. The growing value of c indicates the increase in radial stretch which accelerates the flow along outward direction. But the flow in tangential direction slows down.

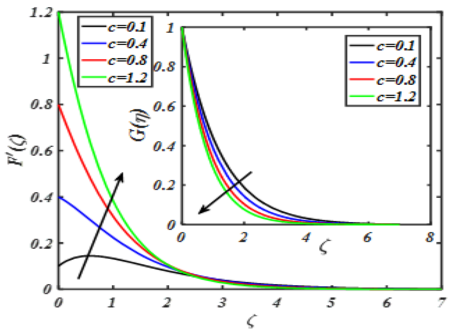

Fig. 1. Variation of F′(ς) and G(ς) under the influence of c.

Fig. 2 shows that increase in c causes reduction in temperature field $\theta(\varsigma)$ and have produces cooling but the concentration function $\phi(\varsigma)$ depicts oscillatory pattern, it increases in the beginning and then decreases to asymtotic state of free stream value.

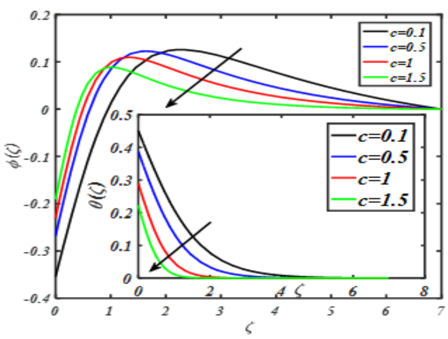

Fig. 2. Variation of φ(ς) and θ(ς) under the influence of c.

Fig. 3 also demonstrate the decreasing pattern of the motile organism density function $\chi(\varsigma)$ when c is incremented.

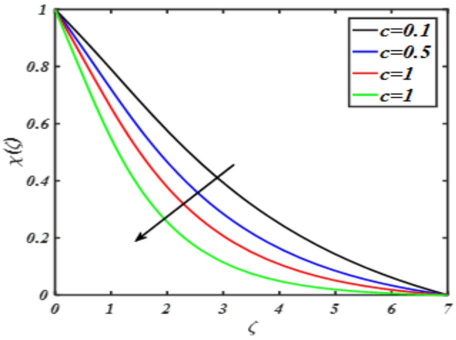

Fig. 3. Variation of χ(ς) under the influence of c.

The increasing values of buoyancy parameter λ_1, caused reduction in flow speed $F'(\varsigma)$ & $G(\varsigma)$ as depicted in Fig. 4 but curve of temperature function $\theta(\varsigma)$ rises up as shown in Fig. 5. Similarly Fig. 6 shows that nano particles concentration $\phi(\varsigma)$ and motile density function $\chi(\varsigma)$ are incremented. This in one of the main investigations of this study.

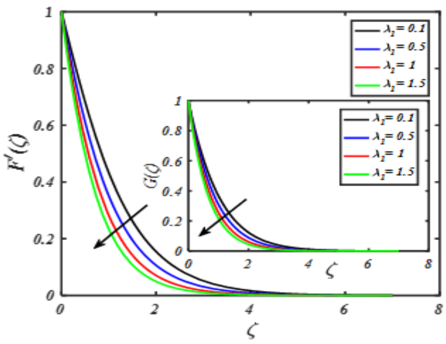

Fig. 4. Variation of F′(ς) and G(ς) under the influence of λ_1.

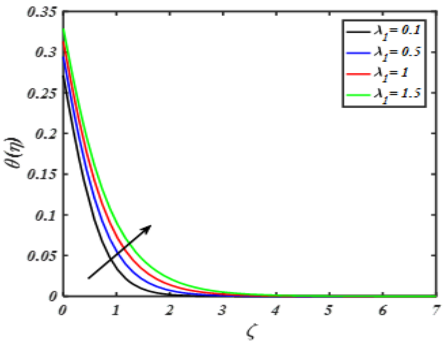

Fig. 5. Variation of θ(ς) under the influence of λ_1.

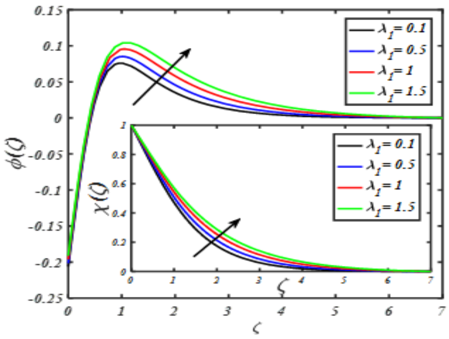

Fig. 6. Variation of φ(ς) and χ(ς) under the influence of λ₁.

The interesting results have been observed with bioconvection due to buoyancy. In this contest other two parameters of interests are Raleigh number N_c and Buoyancy ratio parameter N_r that also emerged due to bioconvection.

It is seen that with increase in N_c and N_r the radial velocity $F'(\varsigma)$ slows down as demonstrated in Fig. 7 and Fig. 8 but on the other hand nano particles concentration $\phi(\varsigma)$ and motile organism density function $\chi(\varsigma)$ both are increased see Figs. 9 and 10.

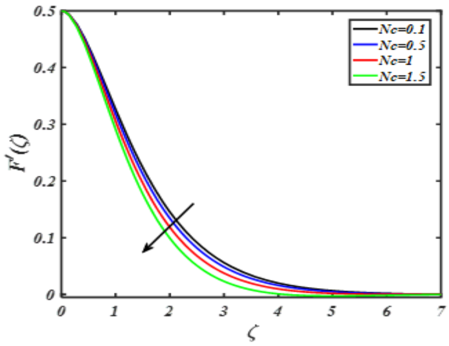

Fig. 7. Variation of F′(ς) under the influence of Nc.

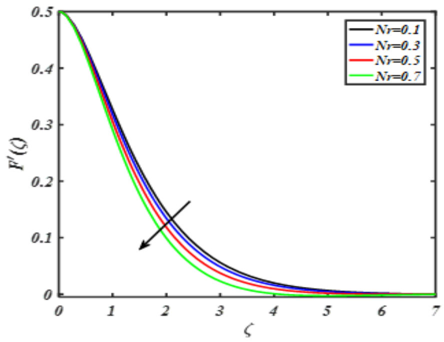

Fig. 8. Variation of F′(ς) under the influence of Nr.

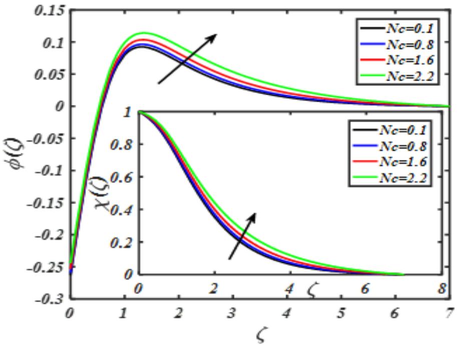

Fig. 9. Variation of φ(ς) and χ(ς) under the influence of Nc.

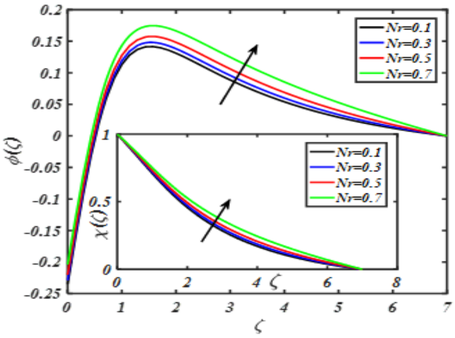

Fig. 10. Variation of φ(ς) and χ(ς) under the influence of N_r.

Fig. 11 indicates the decreasing effect of effective Prandtl number $\Pr_{eff}$ on temperature function $\theta(\varsigma)$ and nano particles concentration $\phi(\varsigma)$. It is because $\Pr_{eff}$ is reciprocal to thermal radiation parameter and thermal conductivity. Hence increase in $\Pr_{eff}$ causes cooling.

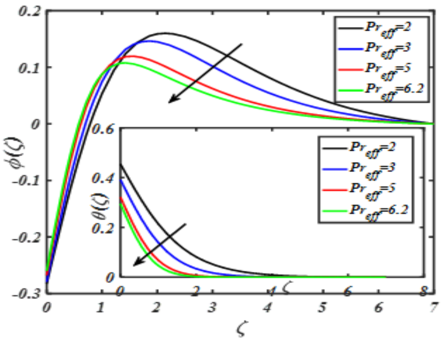

Fig. 11. Variation of φ(ς) and θ(ς) under the influence of Pr_{eff}.

Fig. 12 and Fig. 13 portrait a significant increase in nano particles concentration $\phi(\varsigma)$, temperature and motile organism density with increment in thermopherosis parameter N_t but the nano particle concentration reduces with increment in Brownian motion parameter N_b as presented in Fig. 14.

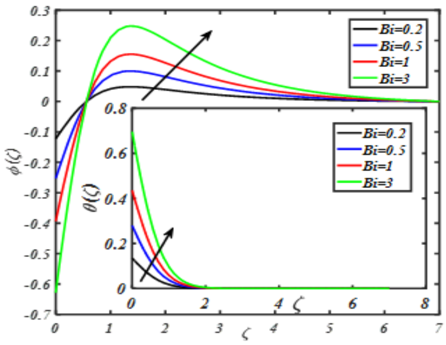

Fig. 12. Variation of φ(ς) and θ(ς) under the influence of Bi.

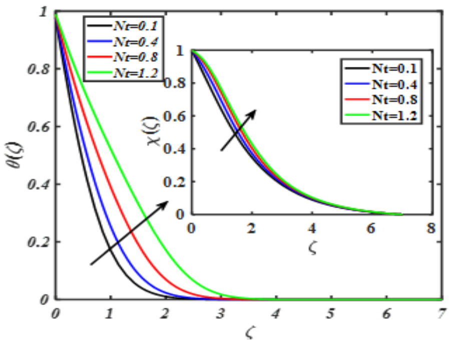

Fig. 13. Variation of χ(ς) and θ(ς) under the influence of Nt.

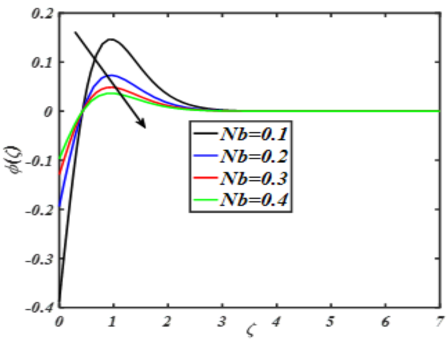

Fig. 14. Variation of φ(ς) under the influence of Nb.

The increase in Biot number which is proportional to heat transfer coefficient marked significant increase in microorganism density function $\chi(\varsigma)$, nano particle concentration $\phi(\varsigma)$, temperature function $\theta(\varsigma)$ as depicted in Fig. 15 and Fig. 16.

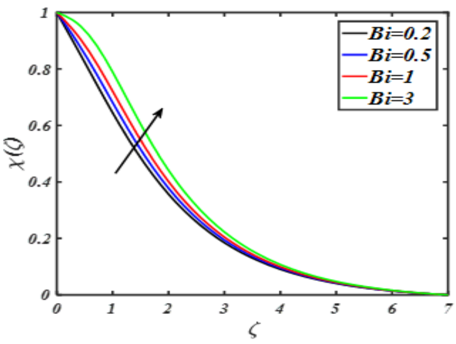

Fig. 15. Variation of χ(ς) under the influence of Bi.

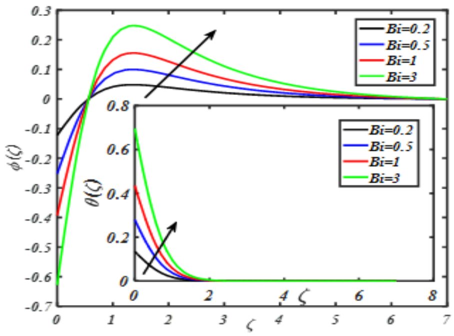

Fig. 16. Variation of φ(ς) and θ(ς) under the influence of Bi.

The increase in Peclet number Pe and microorganism difference parameter δ_1 marked increase in microorganism density function $\chi(\varsigma)$ as shown in Fig. 17.

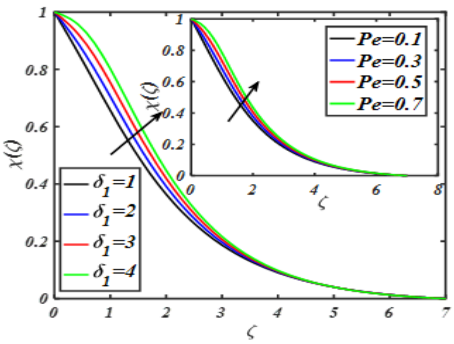

Fig. 17. Variation of χ(ς) under the influence of Pe and δ₁.

Fig. 18 shows that microorganism density function $\chi(\varsigma)$ reduces with increment in BioLevis number L_b Similarly decreasing effect of Schmidt number S_c on $\chi(\varsigma)$ is observed in Fig. 19.

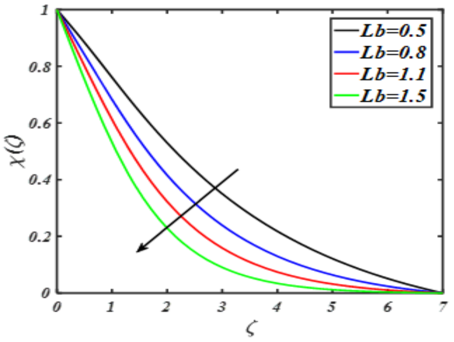

Fig. 18. Variation of χ(ς) under the influence of Lb.

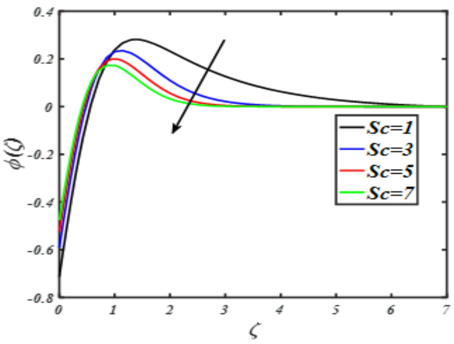

Fig. 19. Variation of φ(ς) under the influence of Sc.

4. Conclusions

Numerical study for bio convection of self- motile microorganism for MHD nano fluid flow over a stretchable rotating disk is disused herein. Matlab coding with bvp4c function is utilized for computational endeavor to bring in to light the efficacy of emerging parameters on physical quantities of interest, to name the nano particle volume fraction $\phi(\varsigma)$, temperature $\theta(\varsigma)$, microorganism density function $\chi(\varsigma)$ and the velocity functions $F'(\varsigma)$ & $G(\varsigma)$. Some major findings are summarized below:

- The growing value of c indicates the increase in radial velocity component $F'(\varsigma)$, but the flow in tangential direction $G(\varsigma)$ slows down. The temperature $\theta(\varsigma)$, motile organism density function $\chi(\varsigma)$ are reduced and nano particle volume fraction $\phi(\varsigma)$ depicts oscillatory pattern.
- The increasing values of buoyancy parameter λ_1, causes reduction in flow speed $F'(\varsigma)$ & $G(\varsigma)$ but $\theta(\varsigma)$ rises up. However, $\phi(\varsigma)$ and $\chi(\varsigma)$ are incremented.
- The increase in Raleigh number N_c and Buoyancy ratio parameter N_r diminished $F'(\varsigma)$ but $\phi(\varsigma)$ and $\chi(\varsigma)$ are incremented.
- The effective Prandtl number $\Pr_{eff}$ marked decreasing impact on $\theta(\varsigma)$ and $\phi(\varsigma)$.
- Thermophoresis parameter N_t made notable increase in $\phi(\varsigma)$, $\theta(\varsigma)$ and $\chi(\varsigma)$.
- Brownian motion parameter N_b reciprocally reduces $\phi(\varsigma)$.
- The incremented Biot number increases $\chi(\varsigma)$, $\phi(\varsigma)$, and $\theta(\varsigma)$.

- The increase in Peclet number Pe and microorganism difference parameter δ_1 marked increase in microorganism density function $\chi(\varsigma)$.
- Microorganism density function $\chi(\varsigma)$ reduces with increments in BioLewis number L_b and Schmidt number S_c.

Acknowledgment

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Abha 61413, Saudi Arabia for funding this work through research groups program under grant number R.G.P-2/88/41.

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