

## Studying of the polarization modes TE and TM for oblique incidence of light on thin films

S. I. Abbas<sup>a,\*</sup>, S. R. Salman<sup>b</sup>, A. S. Abbas<sup>c</sup>

<sup>a</sup>*Al-Karkh University of science, College of science, Medical physics department, Baghdad, Iraq*

<sup>b</sup>*University of Wasit, College of Science, Department of Physics, Kut, Iraq*

<sup>c</sup>*Al-Karkh University of science, College of Geophysic and Remote Sensing, Baghdad, Iraq*

Reflectance, absorbance and phase difference of the polarization modes (s-polarized) and (p-polarized) were described theoretically by using optical multilayers of thin films. A high reflectance layers with very low absorbance at different incident angles ( $0^\circ, 30^\circ, 60^\circ, 80^\circ$ ) can be achieved in the infrared region by adopting design(Air|HL|<sup>4</sup> PbTe). High refractive index PbTe and low refractive index NaCl are used in the infrared region at central wavelength 10  $\mu\text{m}$ . Design (Air|HL|<sup>3</sup> Glass) adopted in the visible region by using TiO<sub>2</sub> and CaF<sub>2</sub> materials at central wavelength 550 nm. Spectral band width of reflectance remain large (7  $\mu\text{m}$ -18  $\mu\text{m}$ ) for s-polarization with increases of the incident angles of the light, but spectral band width of reflectance for p-polarization are decreasing with an increase of the angles where the smallest for incident angle ( $80^\circ$ ) that reach (6.2  $\mu\text{m}$  to 14  $\mu\text{m}$ ) due to increase of the phase shift between two modes of polarization. The absorbance of p-polarization has a larger value than the absorbance of s-polarization mode that causing a decrease in the reflectivity of the TM modes than TE modes.

(Received February 13, 2021; Accepted May 22, 2021)

*Keywords:* Optical coatings, Polarization, Absorption and reflection Spectra, Phase shifting

### 1. Introduction

Optical high reflectors are the most widely used in optical systems are used for solar energy applications, imaging devices, laser cavities and a great deal of work has been done on omnidirectional mirrors [1,2]. Mirrors are mainly divided to the two types: the first type is metallic mirror that reflected a wide range of wavelengths but has absorption that causing loss of light power. The second type is multilayer dielectric mirrors have high reflectivity in a certain range of wavelength, but the reflectivity of these mirrors be affected to the incident angles. The appropriate chosen of the material and the layer thickness of the multilayer dielectric mirrors can be enhanced the reflectivity. A light wave when the electric field vector lies in the incidence plane is known as TM mode (transverse magnetic mode) or called p- polarized light, and a light wave which the electric field vector normal to the incidence plane is called TE (transverse electric mode) or s-polarized. Where s-polarized and p-polarized are derived from the senkrecht perpendicular and German parallel [3-6]. Omnidirectional mirrors reflect the entire incident light wave independent of the incidence angle, within a particular wavelength range [7]. The ability to reflect the light wave from all dielectric thin films of arbitrary angle of incidence that associated with the existence of a photonic gap (1-3), which occur only in a structure with a dielectric function that is periodic along three orthogonal [2]. The directions of the reflected and transmitted components are the same as for the incident light wave. Incident light wave of arbitrary polarization can be split into two components having these simple directions. The reflected and transmitted amplitudes components are possible to calculate for each direction vector separately and then collective.

---

\* Corresponding author: sabahibab@gmail.com

## 2. Theory

The oblique incidence of the light wave at an interface boundary between two optical medium as shown in figure .1. The electric field component parallel to the boundary (p-polarized light) that described by the equation:

$$\mathcal{E}_i \cos \theta_o + \mathcal{E}_r \cos \theta_o = \mathcal{E}_t \cos \theta_1 \quad (1)$$

where  $\mathcal{E}_i, \mathcal{E}_r, \mathcal{E}_t$  incident, reflectance, and transmitted electric field amplitude, and  $\theta_o, \theta_1$  incident and refraction angles. Since  $\mathcal{H} = y\mathcal{E}$ ; where H is the magnetic field amplitude and y is the characteristic admittance of material.

$$y_o \mathcal{E}_i - y_o \mathcal{E}_r = y_1 \mathcal{E}_t \quad (2)$$

$y_o$  is the characteristic admittance of the incident medium,  $y_1$  is the admittance of the first layer. First, eliminate the transmitted electric field  $\mathcal{E}_t$  and then  $\mathcal{E}_r$  from these two equations to obtain:

$$\frac{\mathcal{E}_r}{\mathcal{E}_i} = \frac{y_o \cos \theta_1 - y_1 \cos \theta_o}{y_o \cos \theta_1 + y_1 \cos \theta_o} \quad (3)$$

$$\frac{\mathcal{E}_t}{\mathcal{E}_i} = \frac{2 y_o \cos \theta_o}{y_o \cos \theta_1 + y_1 \cos \theta_o} \quad (4)$$

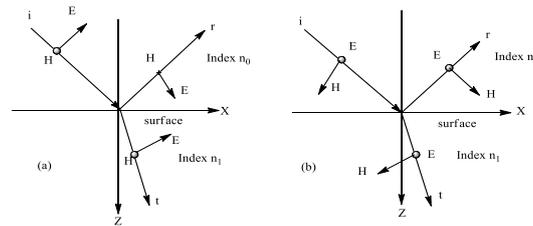


Fig. 1. (a) The positive directions of the electric and magnetic fields for TM mode. (b) The positive directions of the electric and magnetic field for TE mode.

Then simply the two equations that can be written as follows [4]:

$$R = \left( \frac{\mathcal{E}_r}{\mathcal{E}_i} \right)^2 \quad \text{and} \quad T = \frac{y_1}{y_o} \left( \frac{\mathcal{E}_t}{\mathcal{E}_i} \right)^2 \quad (5)$$

The intensity is measured along the wave propagation and the wave transmitted is inclined at an angle that varies from that of the incident wave. The transmittance equation (T) modified to include the angular dependence. In thin films approach the components of the electric field (E) and magnetic field (H) parallel to the boundary that is called tangential components which they express ( $\rho$ ) and ( $\tau$ ) that include amplitudes. The tangential components for electric and magnetic waves are parallel to the boundary of the thin film is have been calculated by using equations (1) and (2). Then can be written:

$$E_i = \mathcal{E}_i \cos \theta_o \quad (6)$$

$$H_i = \mathcal{H}_i = y_o \mathcal{E}_i = \frac{y_o}{\cos \theta_o} E_i \quad (7)$$

$$E_r = \mathcal{E}_r \cos \theta_o \quad (8)$$

$$H_r = \frac{y^\circ}{\cos \theta^\circ} E_r \quad (9)$$

$$E_t = \mathcal{E}_i \cos \theta_1 \quad (10)$$

$$H_t = \frac{y_1}{\cos \theta_1} E_t \quad (11)$$

All these vectors orientations have the same as for incident wave light. The electric and magnetic fields are parallel to the thin film boundary can be written as:

$$E_i + E_r = E_t \quad (12)$$

$$\frac{y^\circ}{\cos \theta^\circ} H_i - \frac{y^\circ}{\cos \theta^\circ} H_r = \frac{y_1}{\cos \theta_1} H_t \quad (13)$$

The amplitude reflection coefficient and amplitude transmission coefficient ( $\rho_p$ ), ( $\tau_p$ ) respectively can be expressed with equations:

$$\rho_p = \frac{E_r}{E_i} = \left( \frac{y^\circ}{\cos \theta^\circ} - \frac{y_1}{\cos \theta_1} \right) / \left( \frac{y^\circ}{\cos \theta^\circ} + \frac{y_1}{\cos \theta_1} \right) \quad (14)$$

$$\tau_p = \frac{E_t}{E_i} = \left( \frac{2y^\circ}{\cos \theta^\circ} \right) / \left( \frac{y^\circ}{\cos \theta^\circ} + \frac{y_1}{\cos \theta_1} \right) \quad (15)$$

The reflectance and the transmittance for p- polarized wave light are written:

$$R_p = \left[ \left( \frac{y^\circ}{\cos \theta^\circ} - \frac{y_1}{\cos \theta_1} \right) / \left( \frac{y^\circ}{\cos \theta^\circ} + \frac{y_1}{\cos \theta_1} \right) \right]^2 \quad (16)$$

$$T_p = \left( \frac{4y^\circ y_1}{\cos \theta^\circ \cos \theta_1} \right) / \left( \frac{y^\circ}{\cos \theta^\circ} + \frac{y_1}{\cos \theta_1} \right)^2 \quad (17)$$

The amplitudes of the electric and magnetic fields components for the s- polarized light waves that parallel to the thin film boundary is given by equations:

$$E_i = \mathcal{E}_i \quad , \quad H_i = \mathcal{H}_i \cos \theta^\circ = (y^\circ \cos \theta^\circ) E_i \quad (18)$$

$$E_r = \mathcal{E}_r \quad , \quad H_r = \mathcal{H}_r \cos \theta^\circ = (y^\circ \cos \theta^\circ) E_r \quad (19)$$

$$E_t = \mathcal{E}_t \quad , \quad H_t = (y_1 \cos \theta_1) E_t \quad (20)$$

Again the tangential vectors components have an orientation is exactly as for normally incident light waves. The amplitude reflection coefficient and amplitude transmission coefficient ( $\rho_s$ ), ( $\tau_s$ ) respectively can be expressed:

$$\rho_s = \frac{E_r}{E_i} = (y^\circ \cos \theta^\circ - y_1 \cos \theta_1) / (y^\circ \cos \theta^\circ + y_1 \cos \theta_1) \quad (21)$$

$$\tau_s = \frac{E_t}{E_i} = (2y^\circ \cos \theta^\circ) / (y^\circ \cos \theta^\circ + y_1 \cos \theta_1) \quad (22)$$

The reflectance and the transmittance equations for s- polarized light are written

$$R_s = [(y^\circ \cos \theta^\circ - y_1 \cos \theta_1) / (y^\circ \cos \theta^\circ + y_1 \cos \theta_1)]^2 \quad (23)$$

$$T_s = (4y^\circ \cos \theta^\circ y_1 \cos \theta_1) / (y^\circ \cos \theta^\circ + y_1 \cos \theta_1)^2 \quad (24)$$

The effective optical thickness ( $n_j d_j \cos \theta_j$ ) is determined by the equation:

$$\delta_j = \frac{2\pi}{\lambda} (n_j d_j \cos \theta_j) \quad (25)$$

where ( $\theta_j$ ) the refracted angle of the light ray, ( $d_j$ ) is the geometrical thickness and ( $n_j$ ) is the refractive index of the layer (j). The refraction angle of the layer is determined by using Snell's law.

$$n_o \sin \theta_o = n_j \sin \theta_j \quad (26)$$

By doing some mathematical simplifications we get:

$$\cos \theta_j = \sqrt{1 - \left( \frac{n_o \sin \theta_o}{n_j} \right)^2} \quad (27)$$

To calculate the p-polarization and s-polarization it is possible to use the following equation [8].

$$\eta = \begin{cases} \frac{n_j}{\cos \theta_j} & p - \text{polarization} \\ n_j \cos \theta_j & s - \text{polarization} \end{cases} \quad (28)$$

$\eta$  is the effective refractive index for two polarization modes.

### 3. Results and discussion

In this research, thin films method is explained with using the theory of optical multilayers for calculating the amplitude of light wave depending on the transmission and reflection coefficients as well as the absorbance and phase shift were measured for TE, transverse electric mode (s-polarized) and TM, for transverse magnetic mode (p-polarized) by using the equation [9].

$$\Phi = \tan^{-1} \left( \frac{i n_o (CB^* - BC^*)}{(n_o^2 BB^* - CC^*)} \right) \quad (29)$$

where  $B$  and  $C$  are total electric and magnetic field amplitudes of the light wave propagating in the layers. Matlab software was used to program the equations of the characteristic matrix that describes the light wave reflectance or absorbance from thin films layers [4]. An alternative route were dependent to determine the optical response of a multilayer thin films stack is so-called transfer matrix approach, which relates the two fields at both interfaces of the layer via a characteristic matrix system [10, 11]. The relative components amplitudes of the incident, refracted, and reflected light waves are depend on their polarization modes with respect to the plane of incidence. Incoming waves can be decomposed into linearly and orthogonally polarized light, with electric components perpendicular and parallel to the incidence plane. High reflectance layers with very low absorbance can be achieved by depending on the dielectric materials have a quarter wave stacks and arranged sequentially low and high refractive index. In this research one design has been adopted, ( $Air|HL|^4 PbTe$ ) which consists of eight layers of a low refractive index (Sodium Chloride,  $n_L = 1.44$  at wavelength  $\lambda_o = 10 \mu m$ ) and a high refractive index (Lead Telluride,  $n_H = 5.64$  at wavelength  $\lambda_o = 10 \mu m$ ). All these films layers will be designed on the PbTe substrate at the central wavelength  $\lambda_o = 10 \mu m$  and for different incident angles ( $0^\circ, 30^\circ, 60^\circ, 80^\circ$ ). In this design two points were considered, the first point in the design is to make the outer layer of a material with a high refractive index to obtain high reflectivity at design wavelength  $\lambda_o = 10 \mu m$ . The second point is to choose a high refractive index ratio ( $n_H/n_L$ )

which is equal (3.91) that causes an increase in the spectra of high reflectance as shown in figures of reflectance versus with wavelength. The largest broadband of reflectance has appeared in the case of normal incidence of light which extends (7  $\mu\text{m}$  to 18  $\mu\text{m}$ ) as shown in Fig. 2.

The oblique incidence of light at angles (30°, 60°, 80°) causes to appear two polarization modes (s-polarization, p-polarization) due to the phase shift between these two modes at different incident angles. Figs. (3, 4, and 5) shows that spectral bandwidth of reflectance remained large (7  $\mu\text{m}$ -18  $\mu\text{m}$ ) for s-polarization or called TE (transverse electric mode) with increasing of the incident angles of the light, but spectral bandwidth of reflectance for p-polarization TM (transverse magnetic mode) is decreasing with an increase of the angles where the smallest for incident angle (80°) that reach (6.2  $\mu\text{m}$  to 14  $\mu\text{m}$ ) due to increase of the phase shift between two modes of polarization. The high reflectance (99.99%) arises when all reflected light waves are of equal phase and which leads to constructive interference at the front surface. To reach high reflectivity with wide broadband for two modes TE and TM when the phase shift between these two modes is equal zero or lower values of the phase shift, and the broadband of high reflectance of TM mode is decreased at high values of incident light that attributes to the high phase shift. The admittance ratio of the s-polarized is increasing with incidence angle of the light, while that for (TM mode) p-admittances is reduced. Since the beam width of the high-reflectance region of a quarter-wave layers decreases with decreasing ratio of these admittances, the beam width of high reflectance region will be less for (TM mode) light than for (TE mode). The fissuring of the optical admittance of thin films layers means also that there is a relative phase shift between p-polarized (TM mode) and s-polarized (TE mode) light occurs when the layers depart from quarter-waves that reflected from high-reflectance multilayers. This effect can be used in the manufacturing phase retarders. The optical admittance ratio of the s-polarized is large, because their splitting increases with incidence angle, and so the corresponding s-reflectance spectrum is high and has a large beam width of the high-reflectance region. The range of beneficial angles of incidence waves will depend slightly on the rate at which the curves of (TM mode) admittance diverge on either side of the intersection of the layer. There are some conditions for varying the phase shift for p-polarization and s-polarization by selecting an overcoat of lower or higher refractive index and altering the thickness of each layer, or by adding different additional layers [12, 13]. The high-reflectance width of a layer with a quarter-wave thickness is a function of the optical admittances ratio for the two materials involved as shown in equation  $[(Y_H/Y_L)_s/(Y_H/Y_L)_p = \cos^2 \vartheta_H/\cos^2 \vartheta_L]$ . This ratio of admittance varies with the incidence angle and is different for s-polarization and p-polarization. The factor  $[\cos^2 \vartheta_H/\cos^2 \vartheta_L]$  is always less than unity so that the high reflectance width for s-polarized mode is always larger than that for p-polarized mode. Within the zone outside the p-polarized mode but inside the (s- mode) high-reflectance zone, the transmittance is high for (p-mode) but low for (s-mode) so that the component acts as a polarizer. The phase shift between TE mode and TM mode polarized light reflected by layers at oblique incidence can be modified by removing or adding different material. There are two effects of oblique incidence angle on the high reflectance coatings. The first case is that the thin film layers appear thinner to the wavelength of light. That can be explained by change in phase induced in the light by the traversal of the thin film. It causes the property of the high reflectance coating to shift towards shorter wavelengths when tilted to oblique incidence. The second case is a shift in the optical admittances of the thin film layers. The optical admittances for TE mode polarization fall and those for TM mode polarization rise. However, the low refractive index layers are larger affected than high refractive index layers, and this means that the ratio of the optical admittance of a high index layer to that of a low index layer of coatings becomes smaller for TM mode polarization but larger for TE mode polarization, implies a weakening characteristic for TM mode polarization and a strengthening for TE mode polarization.

When the thin films layers are designed to be high reflectance coatings, such as a quarter wave stack, the reflectance of s-polarized will be high while the reflectance of p-polarized will be restricted to that at the two outer surfaces where the incident of the light, or emergent, medium makes up one of the materials. Refractive index effects on both optical path length and phase shift but also influences the reflection characteristics at each interface. When designing a thin film

coating, nevertheless the incidence angle and the wavelength of light are usually specified, the refractive index and layers thickness can be varied to optimize the optical performance. Refractive index and thickness of layers are adjusted and these will have an influence on the path length of the light rays within the layer which, in turn, will change the phase values of the propagated light. As the light trip through an optical system, light reflection will occur at the two surfaces which the index alters on either side of the layer. The intensity of reflected light is dependent on the refractive index ratio of the two materials, also depends on the incident angle and polarization of the incident light. When changing the incident angle will lead to change the optical path lengths and the internal angles within each layer, which affect the amount of phase shift in the reflected light beams. At the oblique incidence of light, the two modes s-polarized and p-polarized will reflect differently at each surface of the thin film which will cause differences in the optical performances of the two polarization modes. The maximum reflectance of coatings depends on the refractive index ratio of two materials ( $n_H/n_L$ ) and numbers of periods (N) as shown in the equation [  $R_{max} = [n_m/n_s - (n_H/n_L)^{2N}/n_m/n_s - (n_H/n_L)^{2N}]^2$  ] [8]. The width of the high reflectance region ( $\Delta\lambda_R/\lambda$ ) is greatest when the optical path thickness is equal of quarter-wave stacks and also is dependent on the refractive index ratio of two materials ( $n_H/n_L$ ) as shown in the equation [ $\Delta\lambda_R/\lambda = 4/\pi \sin^{-1}(1 - n_A/n_B/1 + n_A/n_B)$ ]. The optical performance in reflection or transmission of any devices is usually distinguished by polarization P, and the polarization degree can be calculated by using the reflectance values of s- polarized and p-polarized as in the equation; [ $P = [R_P - R_S/R_P + R_S]$ ]. The increase of the incident angle of light which cause to move the Spectrum of reflection towards shorter wavelengths as shown in figures of reflectance. It is possible to observe the absorbance figures versus the wavelength in the infrared range where the absorbance of TM modes is larger values than the absorbance of TE modes for all figures that causing a decrease in the reflectivity values of the TM modes than TE modes.

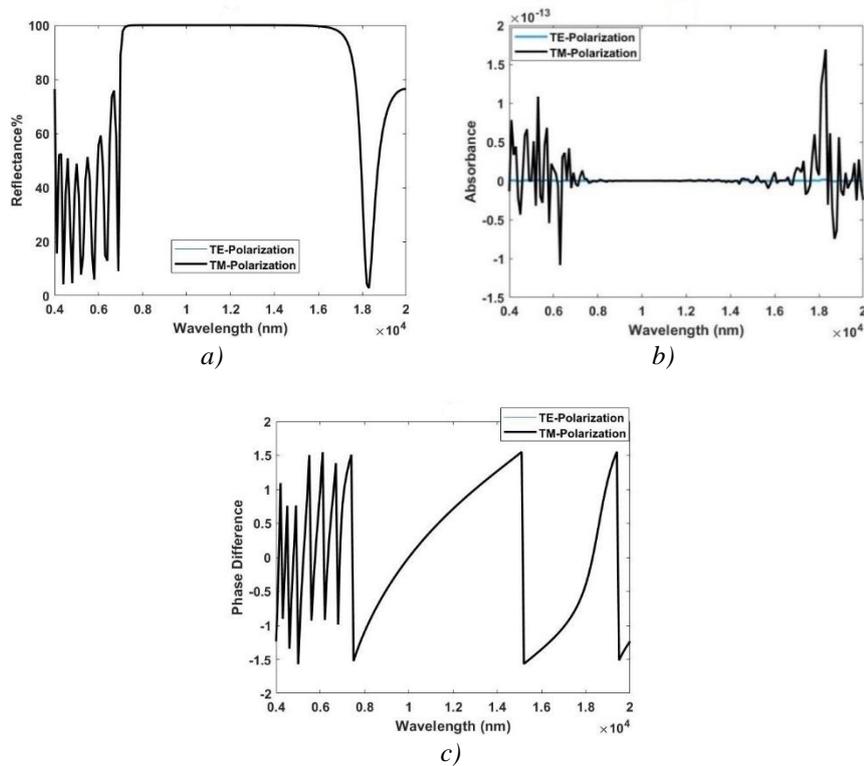


Fig. 2. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength of coatings  $n_H = 5.64$ ,  $n_l = 1.44$ , and  $n_s = 5.64$ , for eight layers at an incident angle ( $0^\circ$ ).

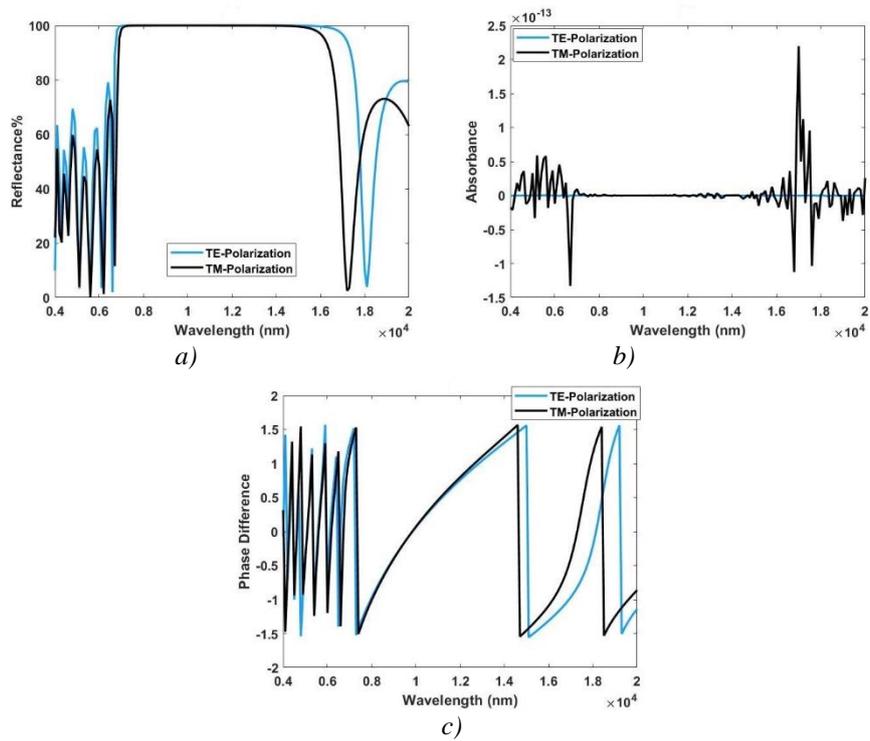


Fig. 3.(a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength for coatings  $n_H = 5.64$ ,  $n_l = 1.44$ , and  $n_s = 5.64$ , for eight layers at an incident angle ( $30^\circ$ ).

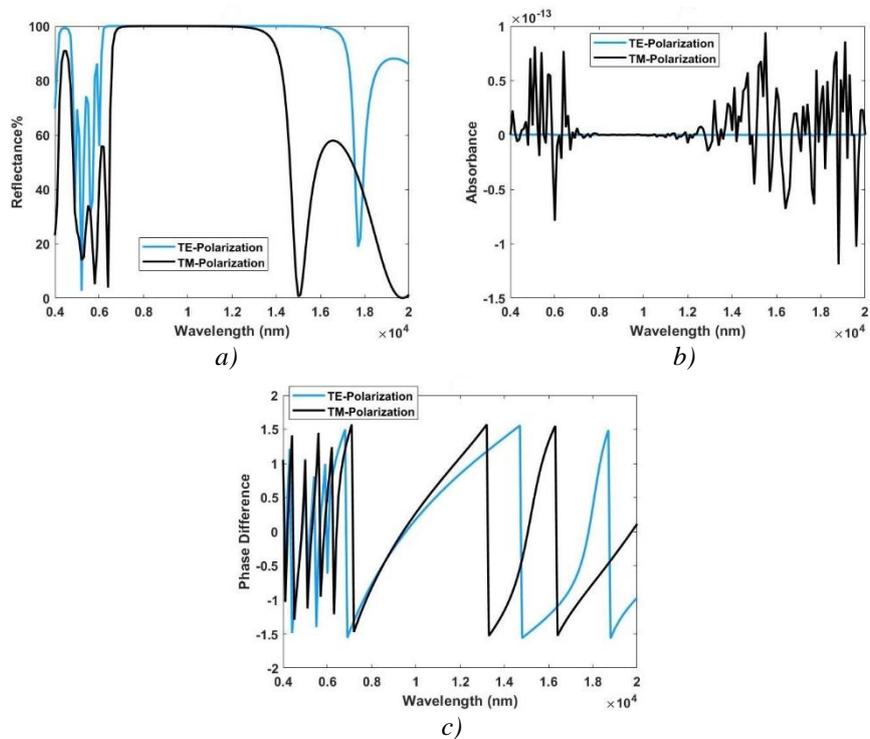


Fig. 4. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength for coatings  $n_H = 5.64$ ,  $n_l = 1.44$ , and  $n_s = 5.64$ , for eight layers at an incident angle ( $60^\circ$ ).

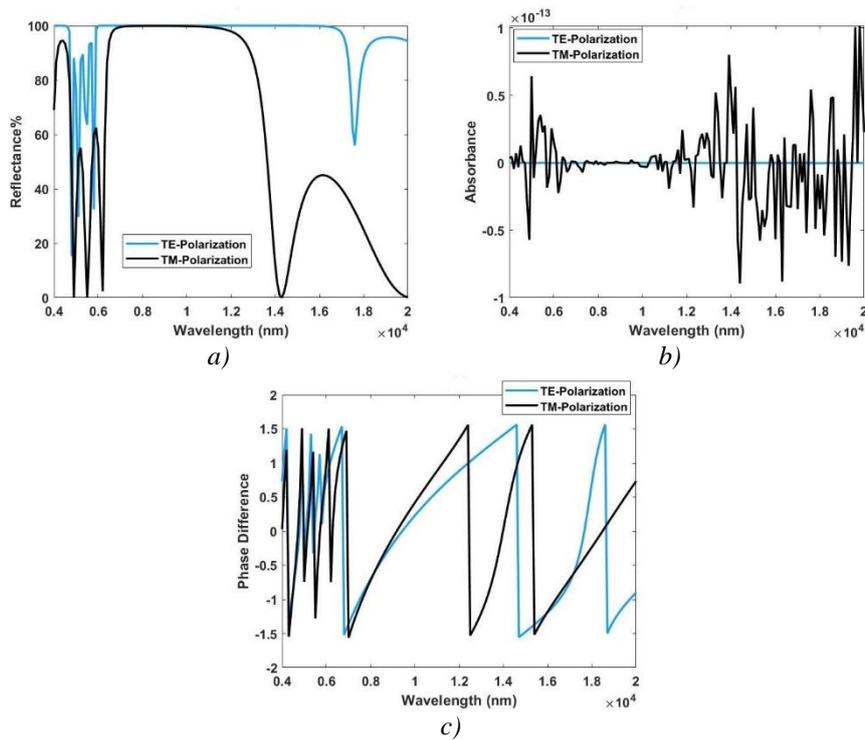


Fig. 5. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength for coatings  $n_H = 5.64$ ,  $n_l = 1.44$ , and  $n_s = 5.64$ , for eight layers at an incident angle ( $80^\circ$ ).

In this research also has been interesting to study the reflectance, absorbance and phase difference versus the wavelength in the visible region (200-1000nm) at a central wavelength (550 nm). Where the high reflectance multilayer was designed as follows ( $Air|HL|^3 Glass$ ) which consists of six layers from a high refractive index ( $n_H = TiO_2 = 2.64$ ), and low refractive index ( $n_L = CaF_2 = 1.43$ ) on the glass substrate. The two modes of reflectance (s-polarized and p-polarized) have appeared when the oblique incidence of light at angles ( $60^\circ, 80^\circ$ ) as shown in figure .7. and figure .8. Where is not appearing at normal incidence of the light as shown in figure .6. It is possible to observe that the reflectance of TM mode polarized is less than TE mode polarized and the maximum reflectance value of TM mode is shifted towards short wavelength. The bandwidth of high reflectance zone has appeared smaller than the bandwidth of high reflectance of the infrared region which is attributed to the small value of the ratio ( $n_H/n_L$ ) which is equal (1.84). Finally, the low refractive index thin films have very small dispersion than of the high refractive index so that can be neglected.

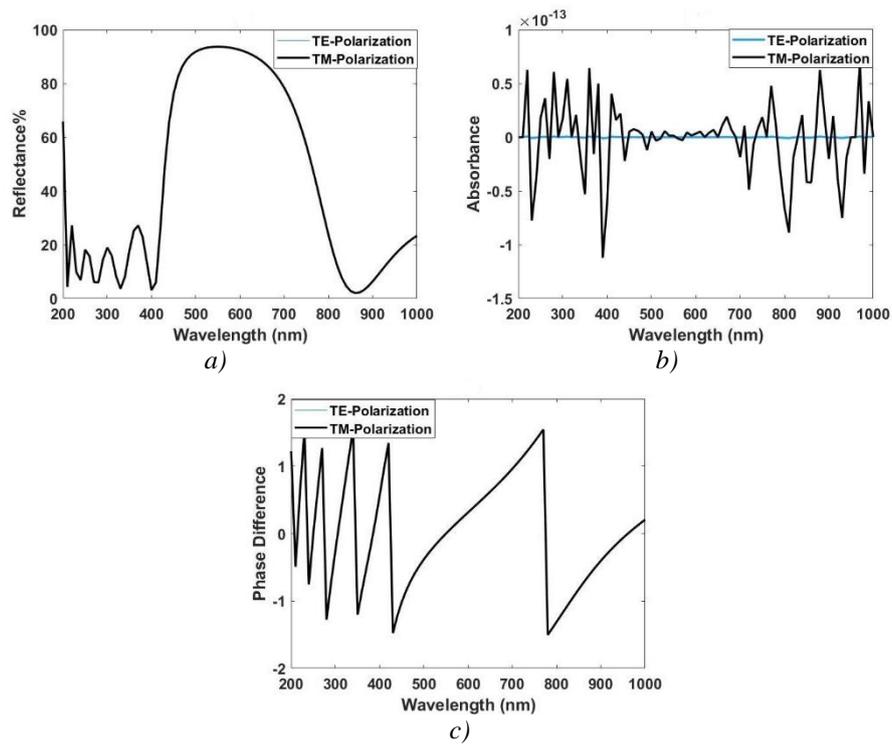


Fig. 6. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength for six layers at an incident angle ( $0^\circ$ ).

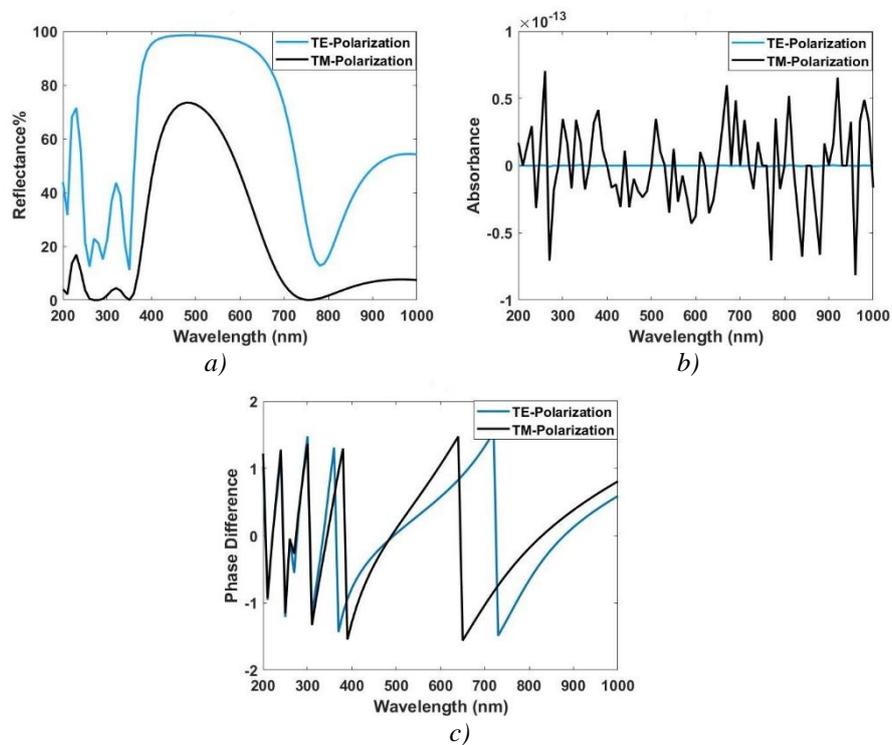


Fig. 7. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength for six layers at an incident angle ( $60^\circ$ ).

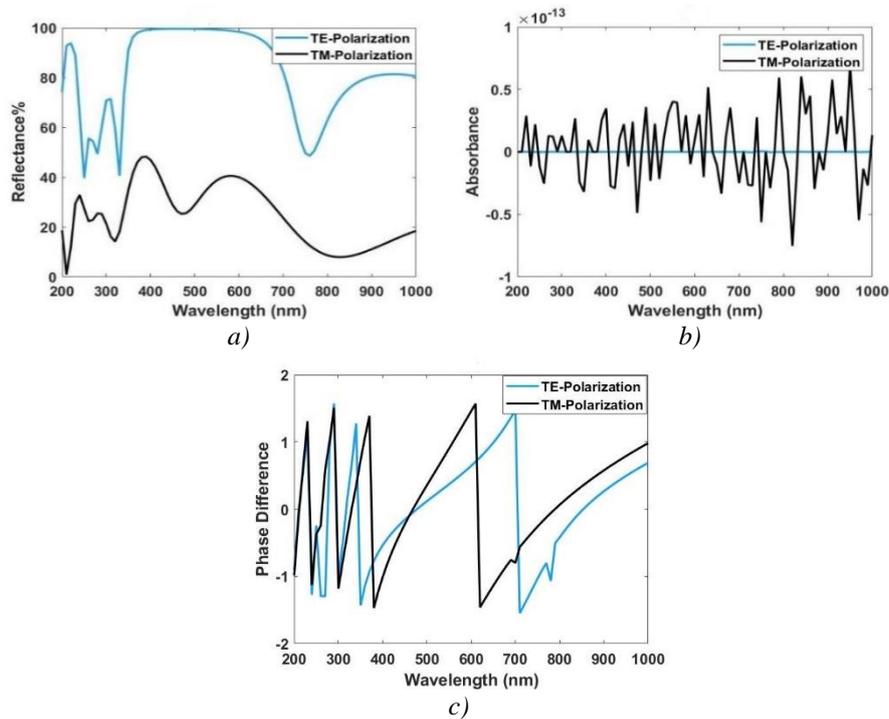


Fig. 8. (a) Reflectance, (b) Absorbance, and (c) Phase Difference as a function of the wavelength at an incident angle ( $80^\circ$ ).

#### 4. Conclusions

Oblique incidence of light generates two polarization modes TM (transverse magnetic mode) and TE (transverse electric mode). The reflectivity of these two modes reaches maximum values with wide broadband when the phase shift between these two modes is equal to zero or lower values of the phase shift, and the broadband of high reflectance of the TM mode decreased at high values of incident light that attributes to the high phase shift. The optical admittance ratio varies with the incidence angle and is different for s-polarization and p-polarization.

The factor  $(\cos^2 \theta_H / \cos^2 \theta_L)$  is always less than unity so that the high reflectance width for s-polarized mode is larger than that for p-polarized mode. The width of the high reflectance region  $(\Delta\lambda_R/\lambda)$  is greatest when the optical path thickness is equal to quarter-wave stacks and also is dependent on the refractive index ratio of the two different materials  $(n_H/n_L)$ . Increase of the incident angle of light which causes to move the spectrum of reflection towards shorter wavelengths. The absorbance of TM modes is larger than the absorbance of TE modes.

#### References

- [1] S. K. Srivastava, S. P. Ojha, Progress In Electromagnetics Research **47**, 181(2007).
- [2] Y. Fink, J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos, E. L. Thomas, Science **282**, 1679 (1998).
- [3] F. L. Pedrotti, L.S. Pedrotti, Introduction to optics”, Prentice-Hall International, New Jersey, (1987).
- [4] H. A. Macleod, Thin- Film Optical Filters, Taylor & Francis Group, (2010).  
file:///C:/Users/sabah/Downloads/9781420073034\_googlepreview.pdf.
- [5] J. P. Pandey, International Journal of Physical Sciences **12**, 137 (2017).
- [6] J. Lekner, Journal of Optics A: Pure and Applied Optics **2**, 349 (2000).
- [7] A. D. A. Flores, L. M. Gaggero, V. Agarwal, Nanoscale research letters **7**, (2012).

- [8] J. A. Dobrowolski, Handbook of Optics, 2<sup>ed</sup>, the Optical Society of America, Mc Graw-Hill, Inc **1**, Ch 42, (1995);  
[www.photonics.intec.ugent.be/education/IVPV/res\\_handbook/v1ch42.pdf](http://www.photonics.intec.ugent.be/education/IVPV/res_handbook/v1ch42.pdf).
- [9] S. I. Abbas, S. R. Salman, S. S. Hashim, Australian journal of basic and applied sciences **11**(11), 186 (2017).
- [10] G. Fowles, Introduction to Modern Optics, 1<sup>st</sup>, Dover Publications, New York, 1990 .
- [11] S. M. Born, E. Wolf, Principles of Optics, 7<sup>th</sup>, Cambridge University Press, Cambridge, (1999), <https://archive.org/details/PrinciplesOfOptics>
- [12] P. B. Clapham, M. J. Downs, R. J. King, Applied Optics **8**(10), 1965 (1968).
- [13] K. Rabinovitch, G. Toker, Proceedings of the Society of Photo-Optical Instrumentation Engineers **2253**, (1994).